PLUS ONE COMPUTER SCIENCE

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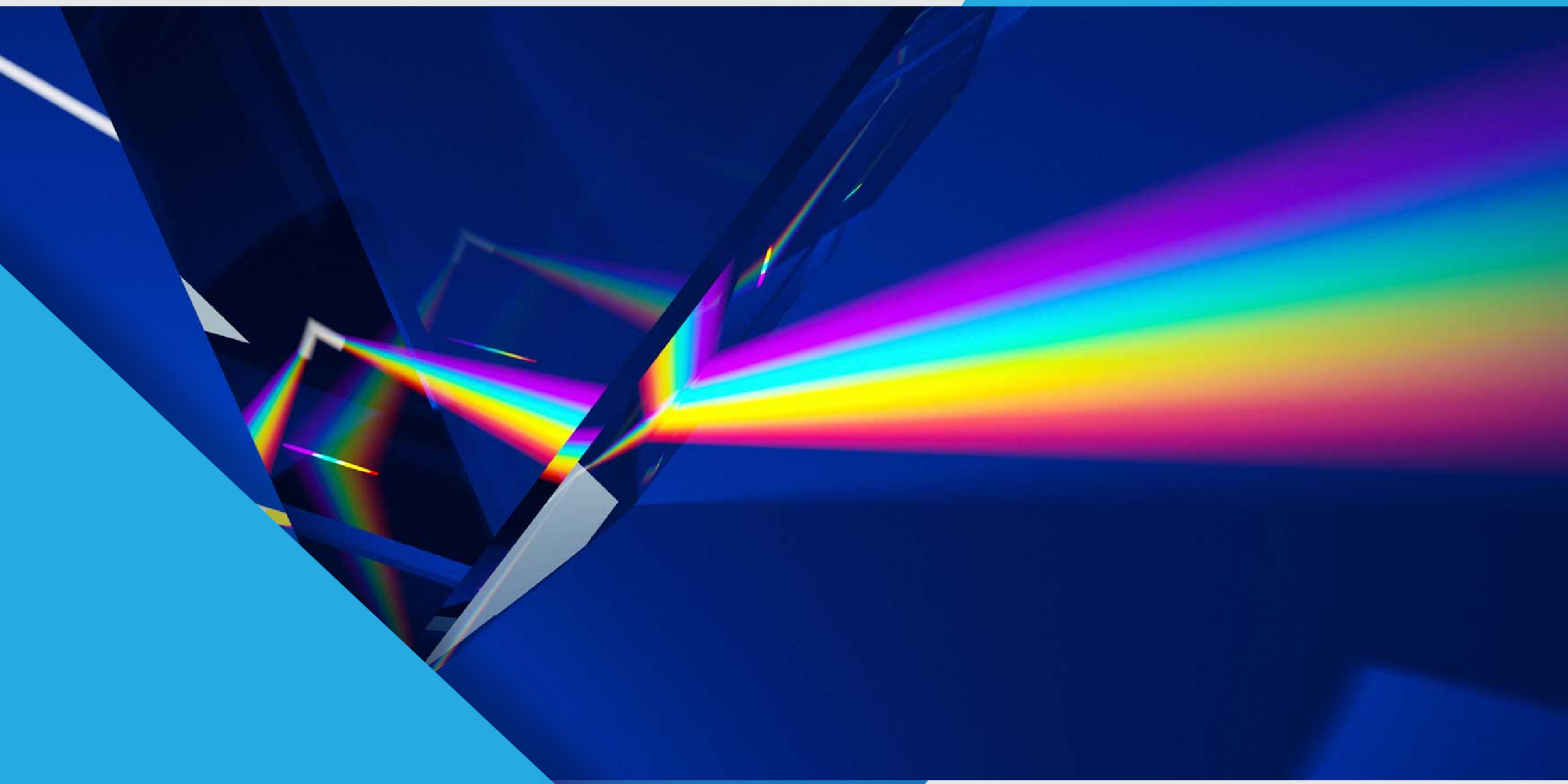
STUDY MATERIAL



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HIGHER SECONDARY NATIONAL SERVICE SCHEME

Chapter 1
Physical World

The word Science originates from the Latin verb **Scientia** meaning **'to know'**
The word Physics comes from a Greek word meaning nature.

Scientific Method

Scientific Method involves the following steps,
1. Systematic observations 2. Controlled experiments 3. Qualitative and quantitative reasoning
4. Mathematical modelling 5. Prediction and verification or falsification of theories

Scope and Excitement of Physics

Basically, there are two domains of interest : **macroscopic and microscopic.**

The macroscopic domain

The macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales. Classical Physics deals mainly with macroscopic phenomena.
The branches of classical physics -Mechanics, Electrodynamics, Optics and Thermodynamics.

- Mechanics founded on Newton's laws of motion and the law of gravitation . It is concerned with the motion of particles, rigid and deformable bodies, and general systems of particles.
- Electrodynamics deals with electric and magnetic phenomena associated with charged and magnetic bodies.
- Optics deals with the phenomena involving light.
- Thermodynamics deals with systems in macroscopic equilibrium and is concerned with changes in internal energy, temperature, entropy, etc., of the system through external work and transfer of heat.

The microscopic domain

The microscopic domain includes atomic, molecular and nuclear phenomena. Classical physics is inadequate to handle this domain and **Quantum Theory** is currently accepted as the proper framework for explaining microscopic phenomena.

Fundamental Forces in Nature

There are four fundamental forces in nature- Gravitational Force, Weak Nuclear Force, Electromagnetic Force and Strong Nuclear Force

Forces	Gravitational Force	Weak Nuclear Force	Electromagnetic Force	Strong Nuclear Force
What ?	It is the force of mutual attraction between any two objects by virtue of their masses	The weak nuclear force appears only in certain nuclear processes such as the β-decay of a nucleus.	Electromagnetic force is the force between charged particles.	The strong nuclear force binds protons and neutrons in a nucleus.
Operate s among	All objects in nature	Some elementary particles, particularly electron and neutrino	Charged particles	Nucleons, heavier elementary particles
Range	Long range force; Infinite	Very short, Sub-nuclear size ($\sim 10^{-16}$m)	Long range force ; Infinite	Short, nuclear size ($\sim 10^{-15}$m)
Nature	Always attractive	Not attractive or repulsive	Similar charges repel and opposite charges attract.	Attractive for distances larger than 0.8 fm and repulsive if they are separated by distances les than 0.8 fm. Force between n-n, p-p, n-p are same. Nuclear force is charge independent.
Relative strength	Weakest force in nature	Stronger than gravitational force , but weaker than electromagnetic force	Stronger than gravitational and weak nuclear force, but weaker than strong nuclear force	Strongest force in nature
Relative strength	10^{-39}	10^{-13}	10^{-2}	1

Nature of Physical Laws

- The physical quantities that remain unchanged in a process are called conserved quantities.
- Some of the general conservation laws in nature include the laws of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc.
- **Some conservation laws are true for one fundamental force but not for the other.**
- Conservation laws have a deep connection with symmetries of nature. Symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.

Chapter 2
Units and Measurement

Fundamental and Derived Quantities

- **The physical quantities, which are independent of each other and cannot be expressed in terms of other physical qualities are called fundamental quantities.**
  Eg: length, mass, time.
- **The physical quantities , which can be expressed in terms of fundamental qualities are called derived quantities.** Eg: volume, velocity, force

Fundamental and Derived Units

- The units for the fundamental or base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as combinations of the base units.
- The units of the derived quantities are called derived units.

Systems of Units

The base units for length, mass and time in these systems were as follows :

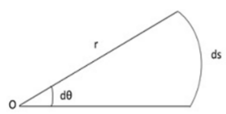
- CGS system - centimetre, gram and second.
- FPS system - foot, pound and second.
- MKS system - metre, kilogram and second.

The International System of Units

In SI system there are seven base units and two supplementary units.

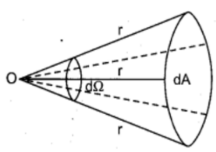
BASE QUANTITY	BASE UNIT	SYMBOL
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

SUPPLEMENTARY QUANTITY	SUPPLEMENTARY UNITS	SYMBOL
Plane Angle	radian	rad
Solid Angle	steradian	sr



$$\text{Angle} = \frac{arc}{radius}$$

$$d\theta = \frac{ds}{r}$$

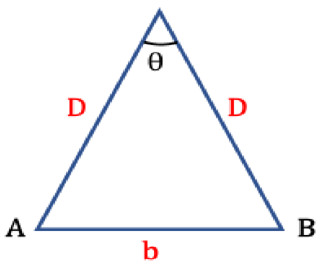


$$\text{Solid angle} = \frac{\text{Intercepted } Area}{Square\ of\ radius}$$

$$d\Omega = \frac{dA}{r^2}$$

Measurement of Large distances-Parallax Method

Large distances such as the distance of a planet or a star from the earth can be measured by parallax method.
To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b.



$$\text{Angle} = \frac{arc}{radius}$$

$$\theta = \frac{b}{D}$$

$$\boxed{D = \frac{b}{\theta}}$$

D=distance of a planet from the earth
b=distance between the observatories
θ= parallax angle

Ranges of Length

We also use certain special length units for short and large lengths.

1 fermi = 1 f = 10^{-15} m
1 angstrom = 1 Å = 10^{-10} m

1 astronomical unit = 1 AU (average distance of the Sun from the Earth)
= 1.496×10^{11} m

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^{8} m s^{-1} in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

Measurement of Mass

While dealing with atoms and molecules, unified atomic mass unit (u), is used.
1 unified atomic mass unit = 1u
= (1/12) of the mass of an atom of carbon-12 isotope ($^{12}_{6}C$) including the mass of electrons

1u = 1.66×10^{-27} kg

The dimensional formulae of some derived quantities

Area - L^2
Density - ML^{-3}
Acceleration - LT^{-2}
Force - MLT^{-2}
Power - ML^2T^{-3}
Pressure - $ML^{-1}T^{-2}$
Volume - L^3
Velocity - LT^{-1}
Momentum - MLT^{-1}
Work or energy - ML^2T^{-2}
Torque - ML^2T^{-2}
Stress - $ML^{-1}T^{-2}$
Modulus of elasticity - $ML^{-1}T^{-2}$

Physical quantities having no dimension and no unit

$$\text{Strain} = \frac{\textit{Change in dimension}}{\textit{Original dimension}} = \frac{[L]}{[L]} = [L^0]$$

$$\text{Relative Density} = \frac{\textit{Density of substance}}{\textit{Density of water}} = \frac{[ML^{-3}]}{[ML^{-3}]} = [L^0]$$

Physical quantities having units, but no dimension

Plane angle
Solid Angle
Angular Displacement

Dimensional analysis and its applications

1) Checking the Dimensional Consistency(correctness) of Equations
2) Deducing Relation among the Physical Quantities

1) Checking the Dimensional Consistency(correctness) of Equations

The principle called the principle of homogeneity of dimensions is used to check the dimensional correctness of an equation.

The principle of homogeneity states that, for an equation to be correct, the dimesions of each terms on both sides of the equation must be the same.

The magnitudes of physical quantities may be added or subtracted only if they have the same dimensions.

If X + Y = Z
[X] = [Y] = [Z]

We cannot add 5m and 10kg
Velocity cannot be added to force.

1.Check the dimensional correctness of the equation s=ut +1/2 at²

$$[s] = L$$
$$[ut] = LT^{-1} \times T = L$$
$$[\tfrac{1}{2}at^2] = LT^{-2} \times T^2 = L$$

Since each term has the same dimension ,this equation is dimensionally correct.

2.Check the dimensional correctness of the equation ½ mv² = mgh

$$[\tfrac{1}{2}mv^2] = M\,[LT^{-1}]^2 = ML^2T^{-2}$$

$$[mgh] = M\;LT^{-2}\;L = ML^2T^{-2}$$

Since each term has the same dimension ,this equation is dimensionally correct.

3.Check the dimensional correctness of the equation E= mc²

$$[E] = ML^2T^{-2}$$

$$[m\,c^2] = M\,[LT^{-1}]^2 = ML^2T^{-2}$$

Since each term has the same dimension ,this equation is dimensionally correct.

2) Deducing Relation among the Physical Quantities

We can deduce relation of a physical quantity which depends upto three physical quantities.

1.Derive the equation for kinetic energy (E) of a body of mass m moving with velocity v

$$E \alpha\; m^x\; v^y$$
$$E = k\,m^x v^y \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M\,L^2\,T^{-2} = M^x\,(LT^{-1})^y$$
$$M^1L^2\,T^{-2} = M^x\,L^y\,T^{-y}$$

equating the dimensions on both sides,

$$x = 1$$
$$y = 2$$

Substituting in eq (1)

$$E = k\,m^1\,v^2$$
$$\boxed{E = k\,mv^2}$$

2) Suppose that the period of oscillation of the simple pendulum depends on its mass of the bob (m), length (l) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

$$T \propto m^x\; l^y\; g^z$$
$$T = k\,m^x\,l^y\,g^z \longrightarrow (1)$$

Writing the dimensions on both sides,

$$M^0L^0T^1 = M^x\,L^y\,(LT^{-2})^z$$
$$M^0L^0T^1 = M^x\,L^y\,L^z\,T^{-2z}$$
$$M^0L^0T^1 = M^x\,L^{y+z}\,T^{-2z}$$

equating the dimensions on both sides,

$$x = 0$$
$$y + z = 0$$
$$-2z = 1 \qquad z = \frac{-1}{2}$$
$$y + \frac{-1}{2} = 0 \qquad y = \frac{1}{2}$$

$$\boxed{T = k m^0\, l^{1/2} g^{-1/2}}$$

$$T = k\frac{l^{1/2}}{g^{1/2}}$$

$$T = k\frac{\sqrt{l}}{\sqrt{g}}$$

$$\boxed{T = k\sqrt{\frac{l}{g}}}$$

Limitations of Dimensional Analysis

1) Dimensional analysis check only the dimensional correctness of an equation, but not the exact correctness.
2) The dimensionless constants cannot be obtained by this method.
3) We cannot deduce a relation, if a physical quantity depends on more than three physical quantities.
4) The method cannot be considered to derive equations involving more than one term
5) A formula containing trigonometric , exponential and logarithmic function can not be derived from it.
6) It does not distinguish between the physical quantities having same dimensions.

Errors In Measurement

Every measurement is approximate due to errors in measurement.

Errors

Systematic errors Random errors.

Systematic errors

The systematic errors are those errors that tend to be in one direction, either positive or negative.

Random errors

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size.

Some of the sources of systematic errors are :

a) Instrumental errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) Imperfection in experimental technique or procedure

(c) Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness

True value or Mean value or Average value

Suppose the values obtained in several measurements are $a_1, a_2, a_3, \dots\dots, a_n$ The arithmetic mean of these values is taken as the True value.

$$a_{mean} = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

Absolute Error

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement.

$$\Delta a_1 = |a_1 - a_{mean}|$$

Mean Absolute Error

The arithmetic mean of all the absolute errors is taken as the final or mean absolute error.

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \cdots + \Delta a_n}{n}$$

Relative Error

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\delta a = \frac{\Delta a_{mean}}{a_{mean}}$$

Percentage Error

Relative error expressed in percent is called Percentage Error.

$$\text{Percentage Error}=\frac{\Delta a_{mean}}{a_{mean}}\ x\ 100\%$$

Example

The centripetal force of a body is given by $F=\frac{mv^2}{r}$. Write an expression for percentage error in centripetal force.

$$\frac{\Delta F}{F}\ x\ 100\% = \frac{\Delta m}{m}\ x\ 100\% + 2\ x\ \frac{\Delta v}{v}\ x\ 100\% + \frac{\Delta r}{r}\ x\ 100\%$$

Significant Figures

The result of measurement is a number that includes all digits in the number that are known reliable plus the first digit that is uncertain.

The reliable digits plus the first uncertain digit in a measurement are known as significant digits or significant figures.

If the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain

Write the number of significant figures in following numbers

0.02380 - 4
23.08 -4
23.80-4
2380-3
43.00-4
4300-2
4.700×10^2 - 4
4.700×10^{-3} -4

Chapter 3
Motion in a Straight Line

Path Length (Distance Travelled)

The total length of the path travelled by an object is called Path Length.

Displacement

Dispalcement is the change in position of the object.

Let x_1 and x_2 be the positions of an object at time t_1 and t_2 .

Displacement, Δx, $= x_2 - x_1$,

Differences between distance (path length) and displacement

1. Distance is a scalar, while displacement is a vector.
2. For a moving particle distance can never be zero or negative while displacement can be zero, positive or negative.
3. For a moving particle, distance can never decrease with time while displacement can. Decrease in displacement with time means that the body is moving towards the initial position.
4. Distance is always greater than or equal to displacement

Average Velocity

Average velocity is defined as the ratio of total displacement to the total time interval.

$$\text{Average velocity} = \frac{\textbf{Total displacement}}{\textbf{Total time interval}}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Average speed

Average speed is defined as the ratio of total path lenth (distance travelled) to the total time interval.

$$\text{Average speed} = \frac{\textbf{Total path length}}{\textbf{Total time interval}}$$

Differences between average speed and average velocity

1. Average speed is a scalar, while average velocity is a vector quantity.
2. For a moving body, speed can never be zero or negative while velocity can be zero, positive or negative.
3. Speed is always greater than or equal to velocity.

Instantaneous velocity

The velocity at an instant is called instantaneous velocity and is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$

Instantaneous speed

Instantaneous speed or simply speed is the magnitude of velocity.

Uniform motion

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in uniform motion along a straight line .

In uniform motion velocity of the object remains constant.

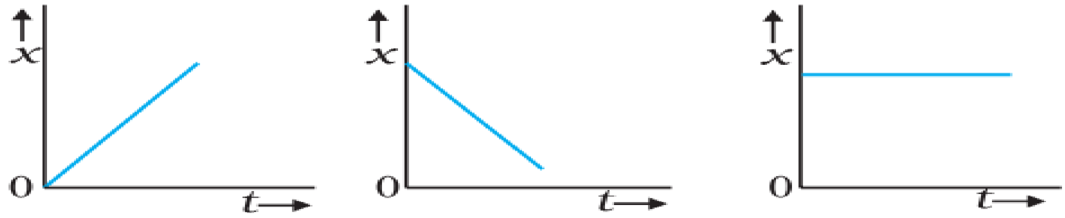
Note that for uniform motion, velocity is the same as the average velocity at all instants.

Position-time graph

Motion of an object can be represented by a position-time graph .For motion along a straight line, say X-axis, only x-coordinate varies with time and we have an x-t graph.

Position-time graph for an object moving with:-

(a) uniform positive (b) uniform negative velocity (c) at rest.

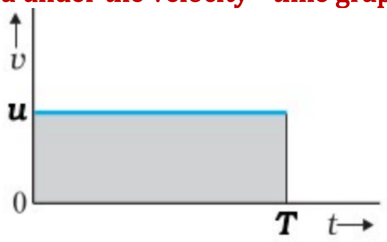


The slope of position-time graph gives the velocity

Velocity - time graph for uniform motion

In uniform motion, velocity is the same at any instant of motion.Therefore, the velocity - time graph is a straight line parallel to the time axis.

The area under the velocity - time graph is equal to the displacement of the particle.



Area = uT=displacement

Acceleration

Suppose the velocity itself is changing with time. In order to describe its effect on the motion of the particle, we require another physical quantity called acceleration. The rate of change of velocity of an object is called acceleration.

Average Acceleration

The average acceleration a over a time interval is defined as the change of velocity divided by the time interval .

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Uniform acceleration

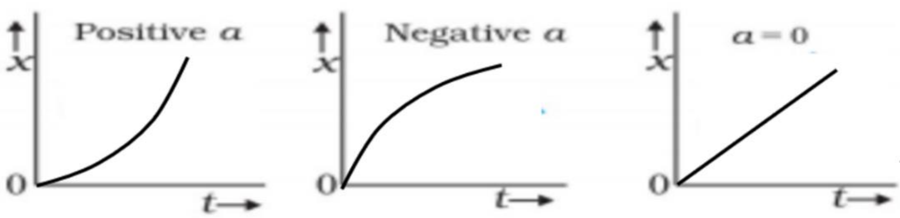
If the velocity of an object changes by equal amounts in equal intervals of time, it has uniform acceleration.

Instantaneous acceleration

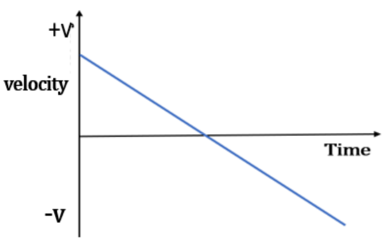
The acceleration of a particle at any instant of its motion is called instantaneous acceleration.

Position-time graph for motion with

(a)positive acceleration (b) negative acceleration c)zero acceleration

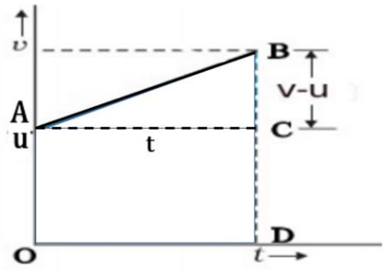


Velocity time graph of a body thrown vertically upwards and returns to ground



Kinematic Equations for Uniformly Accelerated Motion

Consider a body moving with uniform acceleration . The velocity – time graph is as shown in figure



(1) Velocity – time relation

From the graph , acceleration = slope

$$a = \frac{BC}{AC}$$

$$a = \frac{v-u}{t}$$

$$v-u = at$$

$$v = u + at \text{ -------------------- (1)}$$

(2) Position-time relation

Displacement = Area under the graph

$$s = \text{Area of} \square + \text{Area of} \triangle$$

$$s = ut + \tfrac{1}{2}(v-u)\,t$$

But from equation (1)

$$v - u = at$$

$$s = ut + \tfrac{1}{2}\, at \times t$$

$$s = ut + \tfrac{1}{2} at^2 \text{ ------------------(2)}$$

(3)Position – velocity relation

Displacement = Average velocity x time

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{v^2-u^2}{2a}\right)$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as \text{ ----------------(3)}$$

Stopping distance of vehicles

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$v^2 = u^2 + 2as$

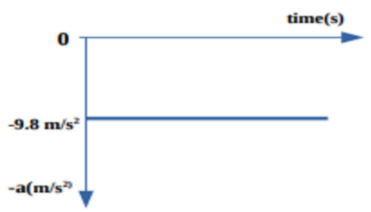
$$0 = u^2 + 2as$$

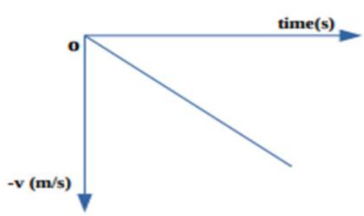
$$-u^2 = 2as$$

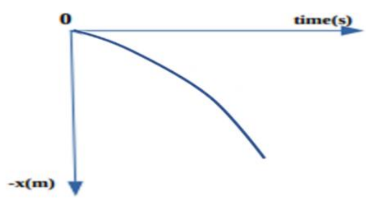
$$s = \frac{-u^2}{2a}$$

Motion of an object under Free Fall

(a)Variation of acceleration with time (b)Variation of velocity with time (c)Variation of distance with tim







Relative Velocity

Consider two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, then

The velocity of object B relative to object A is $v_{BA} = v_B - v_A$

The velocity of object A relative to object B is $v_{AB} = v_A - v_B$

Chapter 4
Motion in a Plane

Scalars and Vectors

A scalar quantity has only magnitude and no direction. It is specified completely by a single number, along with the proper unit.

Eg. distance ,mass , temperature, time .

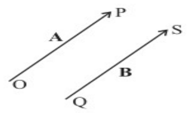
A vector quantity has both magnitude and direction and obeys the triangle law of addition or the parallelogram law of addition. A vector is specified by giving its magnitude by a number and its direction.

Eg.displacement, velocity, acceleration and force.

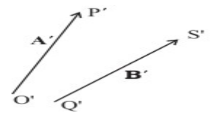
Equality of Vectors

Two vectors A and B are said to be equal if, and only if, they have the same magnitude and the same direction.

(a) Two equal vectors **A** and **B**.



(b) Two vectors **A** ′ and **B** ′ are unequal eventhough they are of same length



Null vector or a Zero vector

A Null vector or a Zero vector is a vector having zero magnitude and is represented by **0** or $\bar{O}$. The result of adding two equal and opposite vectors will be a Zero vector

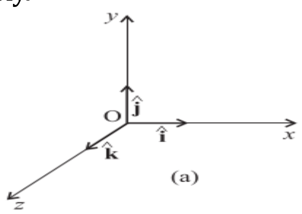
Eg: When a body returns to its initial position its displacement will be a zero vector.

The main properties of $\bar{O}$ are :

$$\bar{A} + \bar{O} = \bar{A}$$
$$\lambda \bar{O} = \bar{O}$$
$$\bar{O}\,\bar{A} = \bar{O}$$

Unit vectors

A unit vector is a vector of unit magnitude and points in a particular direction.
It has no dimension and unit. It is used to specify a direction only.
Unit vectors along the x-, y- and z-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$, respectively.

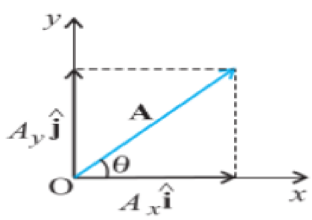


Since these are unit vectors, we have

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

These unit vectors are perpendicular to each other and are called orthogonal unit vectors

Resolution of a vector



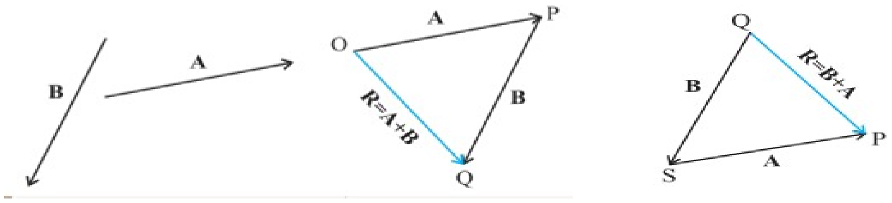
$$\bar{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

where $A_x = A\cos\theta$
$A_y = A\sin\theta$

Addition and Subtraction of Vectors — Graphical Method

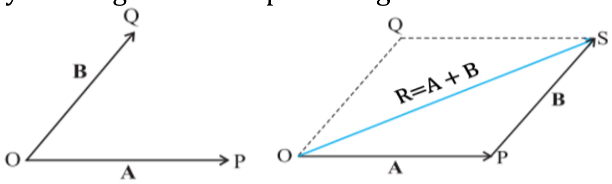
Triangle law of vector addition

If two vectors are represented in magnitude and direction by the two sides of a triangle , their resultant is given by the third side of the triangle.

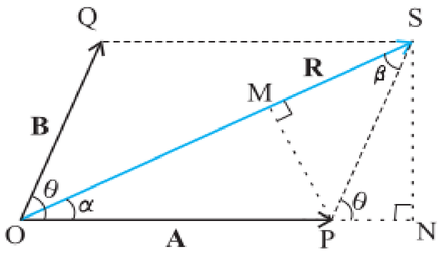


Parallelogram law of vector addition

If two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram ,then their resultant is given by the diagonal of the parallelogram.



Vector Addition – Analytical Method



SN is normal to OP and PM is normal to OS.

ΔSNP , $\cos\theta = PN / PS$ $\qquad \sin\theta = SN / PS$

$\cos\theta = PN / B$ $\qquad \sin\theta = SN / B$

$PN = B\cos\theta$ $\qquad SN = B\sin\theta$

From the geometry of the figure,

$$OS^2 = ON^2 + SN^2$$

$$\text{but } ON = OP + PN$$

$$= A + B\cos\theta$$

$$SN = B\sin\theta$$

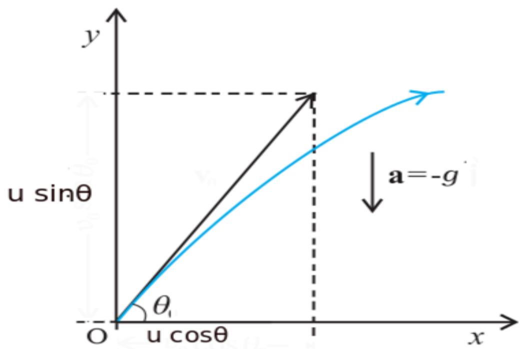
$$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

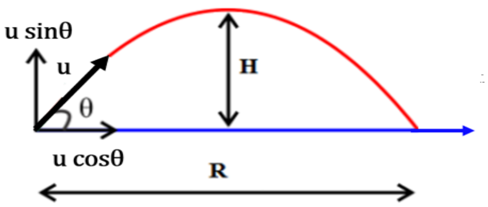
$$\mathbf{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Motion in a Plane-Projectile Motion



- An object that is in flight after being thrown or projected is called a projectile.
- The path (trajectory)of a projectile is a parabole
- The components of initial velocity u are **u cos θ** along horizontal direction and **u sin θ** along vertical direction.
- The x-component of velocity(**u cos θ**) remains constant throughout the motion and hence there is no acceleration in horizontal direction,i.e., $a_x = 0$
- The y- component of velocity (**u sin θ**) changes throughout the motion. At the point of maximum height, **u sin θ** = 0. There is acceleration in horizontal direction, $a_y = -g$

Time of Flight of a projectile (T)



The total time T during which the projectile is in flight is called Time of Flight, T.
Consider the motion in vertical direction,

$$s = ut + \frac{1}{2}at^2$$

$$s=0,\ u = u\sin\theta,\ a = -g,\ t = T$$

$$0 = u\sin\theta\, T - \frac{1}{2}gT^2$$

$$\frac{1}{2}gT^2 = u\sin\theta\, T$$

$$T = \frac{2u\sin\theta}{g}$$

Horizontal range of a projectile (R)

The horizontal distance travelled by a projectile during its time of flight is called the horizontal range.

Horizontal range = Horizontal component of velocity x Time of flight

$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2 \times 2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

R is maximum when sin2θ is maximum, i.e., when θ = 45^0.

$$R_{max} = \frac{u^2}{g}$$

For a given velocity of projection range will be same for angles *θ* and (90-*θ*)

Maximum height of a projectile (H)

It is the maximum height reached by the projectile.
Consider the motion in vertical direction to the highest point

$$v^2 - u^2 = 2as$$

$$u = u\sin\theta,\ v = 0,\ a = -g,\ s = H$$

$$0 - u^2\sin^2\theta = -2gH$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

Uniform Circular Motion

When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**. The word "uniform" refers to the speed, which is uniform (constant) throughout the motion.

Period

The time taken by an object to make one revolution is known as its time period T

Frequency

The number of revolutions made in one second is called its frequency.

$$\upsilon = \frac{1}{T} \qquad \text{unit - hertz (Hz)}$$

Angular velocity (ω)

angular velocity is the time rate of change of angular displacement

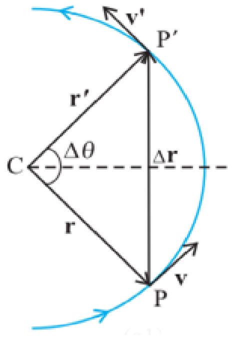
$$\omega = \frac{d\,\theta}{dt}$$

Unit is rad/s

During the time period T ,the angular displacement is 2π radian

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad \omega = 2\pi\,\upsilon$$

Relation connecting angular velocity and linear velocity



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\Delta\,\theta = \frac{\Delta\,r}{r}$$

$$\Delta\,r = r\ \Delta\,\theta$$

Linear velocity $v\ = \frac{\Delta\,r}{\Delta t}$

$$v = \frac{r\,\Delta\,\theta}{\Delta t}$$

But $\omega = \frac{\Delta\,\theta}{\Delta t}$

$$\mathbf{v = r\,\omega}$$

Angular Acceleration

The rate of change of angular velocity is called angular acceleration.

$$\alpha\ = \frac{d\,\boldsymbol{\omega}}{dt}$$

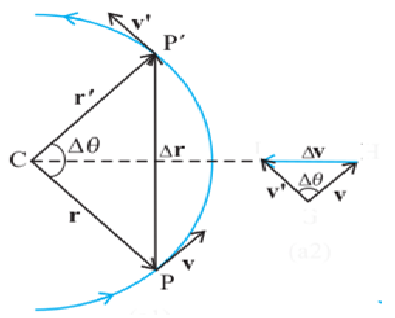
But $\omega = \frac{d\,\theta}{dt}$

$$\alpha = \frac{d}{dt}\left(\frac{d\,\theta}{dt}\right)$$

$$\alpha = \frac{d^2\,\theta}{dt}$$

Centripetal acceleration

A body in uniform circular motion experiences an acceleration , which is directed towards the centre along its radius .This is s called centripetal acceleration .



$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

$$\Delta v = \frac{v \Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta r}{r \Delta t}$$

$$a = \frac{v}{r} \text{ x } r$$

$$a = \frac{v^2}{r}$$

If R is the radius of circular path, then centripetal acceleration .

$$\mathbf{a_c = \frac{v^2}{R}}$$

Example

An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s.

(a) What is the angular speed, and the linear speed of the motion?
(b) Is the acceleration vector a constant vector ? What is its magnitude ?

Period, $T = \frac{100}{7}$ s

(a) The angular speed ω is given by

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{100}{7}} = \frac{2\pi \text{ x}7}{100} = 0.44 \text{ rad/s}$$

The linear speed v is :

$$v = \omega R = 0.44 \times 0.12 = 5.3 \text{ x } 10^{-2} \text{ m s}^{-1}$$

(b) The direction of velocity v is along the tangent to the circle at every point. The acceleration is directed towards the centre of the circle. Since this direction changes continuously, acceleration here is not a constant vector.

$$a = \omega^2 R = (0.44)^2 \text{x} 0.12 = 2.3 \text{x} 10^{-2} \text{ m s}^{-2}$$

Chapter 5
Laws of Motion

Newton's First Law of Motion (Law of inertia)

Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to change that state.

Momentum

Momentum, P of a body is defined to be the product of its mass m and velocity v, and is denoted by p.

$$p = m\,v$$

Momentum is a vector quantity.

Unit = kgm/s

$[p] = ML\,T^{-1}$

Newton's Second Law f Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

$$F \propto \frac{\Delta p}{\Delta t}$$

$$F = \frac{dp}{dt}$$

Why a cricketer draws his hands backwards during a catch?

By Newton's second law of motion ,

$$F = \frac{\Delta p}{\Delta t}$$

When he draws his hands backwards, the time interval (Δt) to stop the ball increases . Then force decreases and it does not hurt his hands.

Force not only depends on the change in momentum but also on how fast the change is brought about.

Derivation of Equation of force from Newton's second law of motion

By Newton's second law of motion ,

$$F = \frac{dp}{dt}$$

For a body of fixed mass m, p=mv

$$F = \frac{d}{dt}mv$$

$$F = m\frac{dv}{dt}$$

$$F = ma$$

Force is a vector quantity

Unit of force is $kgms^{-2}$ or newton (N)

Impulse (I)

Impulse is the the product of force and time duration, which is the change in momentum of the body.

Impulse = Force × time duration

$$I = F\,x\,t$$

Unit = $kg\,m\,s^{-1}$ $[I] = M\,L\,T^{-1}$

Impulsive force

A large force acting for a short time to produce a finite change in momentum is called an impulsive force.

Example: a ball hits a wall and bounces back.

Impulse momentum Principle

Impulse is equal to the change in momentum of the body.

By Newton's second law of motion,

$$F = \frac{dp}{dt}$$

$$F\,x\,dt = dp$$

$$I = dp$$

Impulse = change in momentum

Example

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s^{-1} . If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball.

Impulse = change of momentum

Change in momentum = final momentum – initial momentum

Change in momentum = $0.15 \times 12 - (0.15 \times -12)$

Impulse = 3.6 N s

Newton's Third Law of Motion

To every action, there is always an equal and opposite reaction.

- Action and reaction forces act on different bodies, not on the same body. So they do not cancel each other , eventhough they are equal and opposite.

Law of Conservation of Momentum

The total momentum of an isolated system of interacting particles is conserved.

Or

When there is no external force acting on a system of particles ,their total momentum remains constant.

Proof of law of conservation of momentum

By Newton's second law of motion , $F=\frac{dp}{dt}$

When $F = 0$

$$\frac{dp}{dt} = 0$$

$$dp = 0 ,$$

$$p=\text{constant}$$

Thus when there is no external force acting on a system of particles, their total momentum remains constant.

Applications of law of conservation of linear momentum

1.Recoil of gun

When a bullet is fired from a gun , the backward movement of gun is called recoil of the gun.

If p_b and p_g are the momenta of the bullet and gun after firing

$$p_b + p_g = 0$$

$$p_b = - p_g$$

The negative sign shows that the gun recoils to conserve momentum.

Expression for Recoil velocity and muzzle velocity

Momentum of bullet after firing , $p_b = mv$

Recoil momentum of the gun after firing , $p_g = MV$

$$p_b = - p_g$$

$$mv = -MV$$

Recoil velocity of gun , $V=\frac{-mv}{M}$

Muzzle velocity of bullet , $v=\frac{-MV}{m}$

M= mass of gun, V= recoil velocity of bullet

m= mass of bullet, v=muzzle velocity of bullet

2. The collision of two bodies

By Newton's second law , $F = \frac{\Delta P}{\Delta t}$

$$F\Delta t = \Delta P$$

F_{AB} changes the momentum of body A

$$F_{AB}\, \Delta t = p'_A - p_A \text{-----------------}(1)$$

F_{BA} changes the momentum of body B

$$F_{BA}\, \Delta t = p'_B - p_B \text{------------}(2)$$

By Newton's third law

$$F_{AB} = -F_{BA} \qquad \text{-----------}(3)$$

$$p'_A - p_A = -(p'_B - p_B)$$

$$p'_A + p'_B = p_A + p_B$$

Total Final momentum = Total initial momentum

i.e. , the total final momentum of the isolated system equals its total initial momentum.

Common Forces in Mechanics

There are two types of forces in mechanics- Contact forces and Non contact forces.

Contact forces

A contact force on an object arises due to contact with some other object: solid or fluid.

Eg: Frictional force, viscous force, air resistance

Non contact forces

A non contact force can act at a distance without the need of any intervening medium.

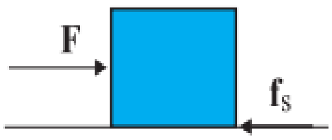
Eg: Gravitational force.

Friction

The force that opposes (impending or actual) relative motion between two surfaces in contact is called frictional force.

There are two types of friction-Static and Kinetic friction

Static friction f_s



Static friction is the frictional force that acts between two surfaces in contact before the actual relative motion starts. Or Static friction f_s opposes impending relative motion.

- The maximum value of static friction is $(f_s)_{max}$
- The limiting value of static friction $(f_s)_{max}$, is independent of the area of contact.
- The limiting value of static friction $(f_s)_{max}$, varies with the normal force(N)

$$(f_s)_{max} \,\alpha N$$

$$(f_s)_{max} = \mu_s N$$

Where the **constant μ_s is called the coefficient of static friction** and depends only on the nature of the surfaces in contact.

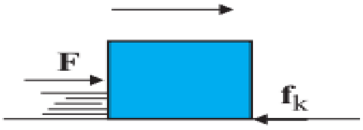
The Law of Static Friction

The law of static friction may thus be written as , $f_s \le \mu_s N$

Or

$$(f_s)_{max} = \mu_s N$$

Kinetic friction f_k



Frictional force that opposes (actual) relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by f_k. .

- Kinetic friction is independent of the area of contact.
- Kinetic friction is nearly independent of the velocity.
- Kinetic friction , f_k varies with the normal force(N)

$$f_k \propto N$$

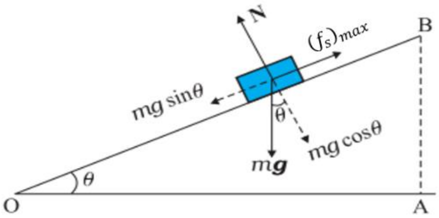
$$\mathbf{f_k = \mu_k N}$$

where μ_k the **coefficient of kinetic friction**, depends only on the surfaces in contact.
μ_k is less than μ_s

The Law of Kinetic Friction

The law of kinetic friction can be written as , $\mathbf{f_k = \mu_k N}$
where μ_k the **coefficient of kinetic friction**,

Body on an inclined surface



The forces acting on a block of mass m When it just begins to slide are
(i) the weight mg
(ii) the normal force N
(iii) the maximum static frictional force $\mathbf{(f_s)_{max}}$

In equilibrium, the resultant of these forces must be zero.

$$m\,g \sin\theta = (f_s)_{max}$$

$$\text{But } (f_s)_{max} = \mu_s N$$

$$mg \sin\theta = \mu_s N \text{------------}(1)$$

$$m\,g \cos\theta = N \text{-------------}(2)$$

$$\text{Eqn}\frac{(1)}{(2)} \text{ --------- } \frac{mg\sin\theta}{m\,g\cos\theta} = \frac{\mu_s N}{N}$$

$$\mathbf{\mu_s = \tan\theta}$$

This angle whose tangent gives the coefficient of friction is called angle of friction.

Rolling Friction

It is the frictional force that acts between the surfaces in contact when one body rolls over the other.
Rolling friction is much smaller than static or sliding friction

Disadvantages of friction

Friction is undesirable in many situations, like in a machine with different moving parts, friction opposes relative motion and thereby dissipates power in the form of heat, etc. Friction produces wear and tear.

Advantages of friction

In many practical situations friction is critically needed. Kinetic friction is made use of by brakes in machines and automobiles. We are able to walk because of static friction. It is impossible for a car to move on a very slippery road. On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

Methods to reduce friction

(1)Lubricants are a way of reducing kinetic friction in a machine.
(2)Another way is to use ball bearings between two moving parts of a machine.
(3) A thin cushion of air maintained between solid surfaces in relative motion is an effective way of reducing friction.

Circular Motion

The acceleration of a body moving in a circular path is directed towards the centre and is called centripetal acceleration.

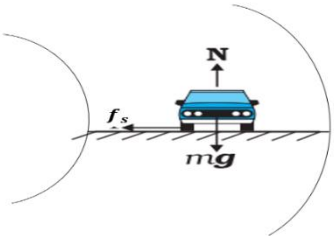
$$\mathbf{a = \frac{v^2}{R}}$$

The force f providing centripetal acceleration is called the centripetal force and is directed towards the centre of the circle.

$$\mathbf{f_s = \frac{mv^2}{R}}$$

where m is the mass of the body, R is the radius of circle.

Motion of a car on a curved level road



Three forces act on the car.

(i) The weight of the car, mg
(ii) Normal reaction, N
(iii) Frictional force, f_s

As there is no acceleration in the vertical direction

$$N = mg$$

The static friction provides the centripetal acceleration

$$f_s = \frac{mv^2}{R}$$

$$\text{But},\ f_s \le \mu_s N$$

$$\frac{mv^2}{R} \le \mu_s mg \quad (N=mg)$$

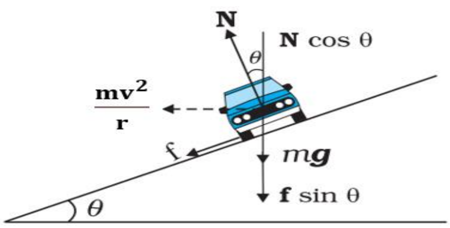
$$\mathbf{v^2 \le \mu_s Rg}$$

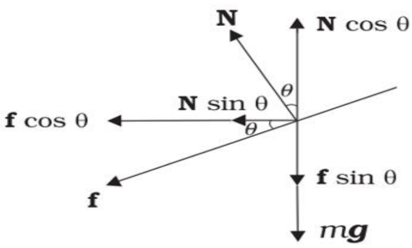
$$\mathbf{v_{max} = \sqrt{\mu_s Rg}}$$

This is the maximum safe speed of the car on a circular level road.

Motion of a car on a banked road

Raising the outer edge of a curved road above the inner edge is called banking of curved roads.





$$N\cos\theta = mg + f\sin\theta$$

$$N\cos\theta - f\sin\theta = mg \text{ --------------(1)}$$

The centripetal force is provided by the horizontal components of N and f_s.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \text{ -------------(2)}$$

$$\frac{\text{Eqn(1)}}{\text{Eqn(2)}} \text{ -----} \frac{N\cos\theta - f\sin\theta}{N\sin\theta + f\cos\theta} = \frac{mg}{\frac{mv^2}{R}}$$

Dividing throughout by $N\cos\theta$

$$\frac{1 - \frac{f}{N}\tan\theta}{\tan\theta + \frac{f}{N}} = \frac{Rg}{v^2}$$

But, $\frac{f}{N} = \mu_s$ for maximum speed

$$\frac{1 - \mu_s\tan\theta}{\tan\theta + \mu_s} = \frac{Rg}{v^2}$$

$$v^2 = \frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}$$

$$\mathbf{v_{max} = \sqrt{\frac{Rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}}}$$

This is the maximum safe speed of a vehicle on a banked curved road.

If friction is absent, $\mu_s = 0$

Then Optimum speed, $\mathbf{v_{optimum} = \sqrt{Rg\tan\theta}}$

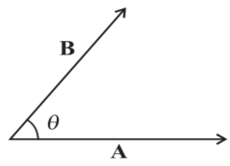
Chapter 6
Work ,Energy and Power

The Scalar Product or Dot Product

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}.\vec{B}$ (read A dot B) is defined as

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

where θ is the angle between the two vectors

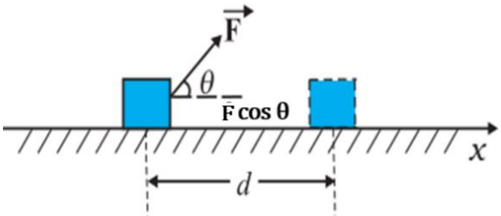


Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction.

- For unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ we have
  $$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$
  $$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$$

- If $\vec{A}$ and $\vec{B}$ are perpendicular
  $$\vec{A}\cdot\vec{B} = A\,B\cos 90 = 0$$

Work



The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.

$$W = (F\cos\theta)d$$
$$W = F\,d\cos\theta$$
$$W = \vec{F}\cdot\vec{d}$$

Work can be zero, positive or negative.

Zero Work

The work can be zero,if

(i)the displacement is zero .

When you push hard against a rigid brick wall, the force you exert on the wall does no work.

A weightlifter holding a 150 kg mass steadily on his shoulder for 30 s does no work on the load during this time.

(ii) the force is zero.

A block moving on a smooth horizontal table is not acted upon by a horizontal force (since there is no friction), but may undergo a large displacement.

(iii) the force and displacement are mutually perpendicular

Here $\theta = 90^\circ$, cos (90) = 0.

For the block moving on a smooth horizontal table, the gravitational force mg does no work since it acts at right angles to the displacement.

Positive Work

If θ is between 0° and 90°, cos θ is positive and work positive.

Eg: Workdone by Gravitational force on a freely falling body is positive

Negative work

If θ is between 90° and 180°, cos θ is negative and work negative.

Eg: the frictional force opposes displacement and $\theta = 180^\circ$.

Then the work done by friction is negative ($\cos 180^\circ = -1$).

Units of Work and Energy

- Work and Energy are scalar quantities.
- Work and energy have the same dimensions, $[ML^2T^{-2}]$.
- The SI unit is **kgm^2s^{-2}** or **joule (J)**, named after the famous British physicist James Prescott Joule.

Alternative Units of Work/Energy in J

erg	10^{-7} J
electron volt (eV)	1.6×10^{-19} J
calorie (cal)	4.186 J
kilowatt hour (kWh)	3.6×10^{6} J

Kinetic Energy

The kinetic energy is the energy possessed by a body by virtue of its motion.

If an object of mass m has velocity v, its kinetic energy K is

$$K = \frac{1}{2}m\bar{v}\cdot\bar{v} = \frac{1}{2}mv^2$$

Kinetic energy is a scalar quantity.

The Work-Energy Theorem

The work-energy theorem can be stated as :**The change in kinetic energy of a particle is equal to the work done on it by the net force.**

Proof

For uniformly accelerated motion

$$v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we have

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs$$

$$K_f - K_i = W$$

Change in KE = Work

Potential Energy

Potential energy is the 'stored energy' by virtue of the position or configuration of a body.

- A body at a height h above the surface of earth possesses potential energy due to its position.
- A Stretched or compressed spring possesses potential energy due to its state of strain.

Gravitational potential energy of a body of mass m at a height h above the surface of earth is mgh.

Gravitational Potential Energy , V =mgh

Conservative Force

A force is said to be conservative, if it can be derived from a scalar quantity.

$$F = \frac{-dV}{dx} \text{ where V is a scalar}$$

Eg: Gravitational force, Spring force.

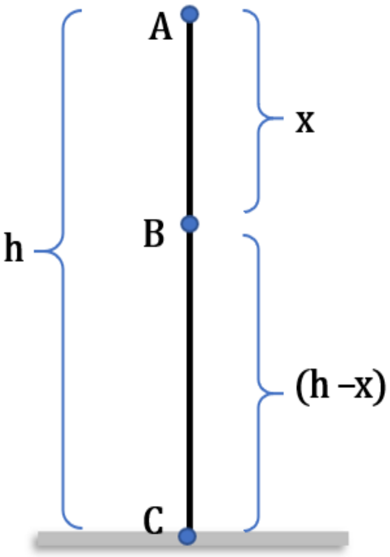
- The work done by a conservative force depends only upon initial and final positions of the body
- The work done by a conservative force in a cyclic process is zero

Note: Frictional force , air resistance are non conservative forces.

The Conservation of Mechanical Energy

The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

Conservation of Mechanical Energy for a Freely Falling Body



At Point A

PE = mgh

KE = 0 (since v=0)

TE = PE + KE

= mgh + 0

TE = mgh----------(1)

At Point B

PE = mg (h-x)

KE = ½ mv^2

$v^2 = 2gx$

KE = ½ m x 2gx

KE= mgx

TE = PE + KE

TE = mg (h-x) + mgx

TE = mgh-------------(2)

At Po int C

PE = 0 (Since h=0)

KE = ½ mv^2

$v^2 = 2gh$

KE = ½ m x 2gh

KE= mgh

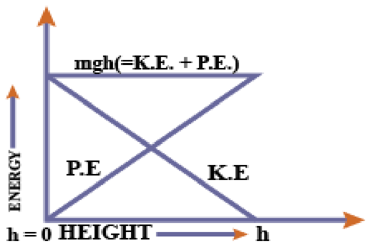
TE = PE + KE

TE = 0 + mgh

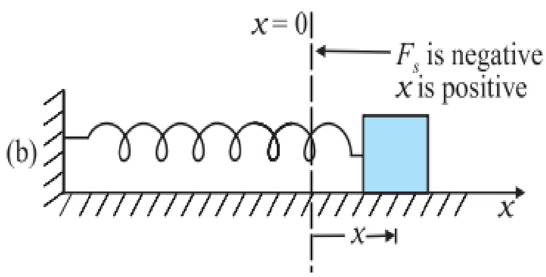
TE = mgh-------------(3)

From eqns (1), (2) and (3), it is clear that the total mechanical energy is conserved during the free fall.

Graphical variation of KE and PE with height from ground



The Potential Energy of a Spring



The the spring force $\mathbf{F = -kx}$

The work done by the spring force is

$$W = \int_0^x F\,dx$$

$$W = -\int_0^x kx\,dx$$

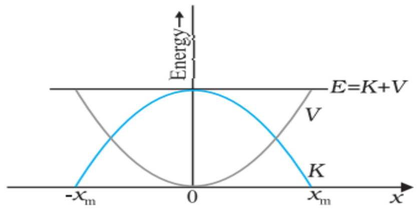
$$W = -\frac{1}{2}kx^2$$

his work is stored as potential energy of spring

$$\mathbf{PE = \frac{1}{2}kx^2}$$

- **At equilibrium position PE is zero and KE is max.**
- **At extreme ends, the PE is maximum and KE is zero.**
- The kinetic energy gets converted to potential energy and vice versa, however, the total mechanical energy remains constant.

Graphical variation of kinetic Energy and potential of a spring



The Equivalence of Mass and Energy

Mass and energy are equivalent and are related by the relation

$$E = m\,c^2$$

This is called Einstein's mass energy relation.

where c, the speed of light in vacuum is approximately $3 \times 10^8 \text{m}\, s^{-1}$.

The Principle of Conservation of Energy

Energy can neither be created, nor destroyed. Energy may be transformed from one form to another but the total energy of an isolated system remains constant.

Power

Power is defined as the time rate at which work is done or energy is transferred.
The average power of a force is defined as the ratio of the work, W, to the total time t taken.

$$P_{av} = \frac{W}{t}$$

The instantaneous power

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dW}{dt}$$

The work done, dW = F. dr.

$$P = F\,.\frac{dr}{dt}$$

$$P = F\,.\,v$$

- SI unit of power is called a watt (W). 1W = 1 J/s
- Another unit of power is the horse-power (hp) . **1 hp = 746 W**
  This unit is still used to describe the output of automobiles, motorbikes, etc

kilowatt hour

Electrical energy is measured in kilowatt hour (kWh).

$$1\text{kWh} = 3.6 \times 10^6\ \text{J}$$

Collisions

In all collisions the total linear momentum is conserved; the initial momentum of the system is equal to the final momentum of the system. There are two types of collisions Elastic and Inelastic.

Elastic Collisions

The collisions in which both linear momentum and kinetic energy are conserved are called elastic collisions.
Eg: Collision between sub atomic particles

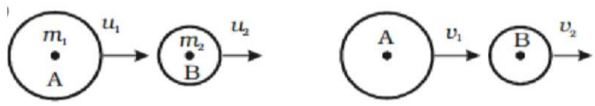
Inelastic Collisions

The collisions in which linear momentum is conserved, but kinetic energy is not conserved are called inelastic collisions. . Part of the initial kinetic energy is transformed into other forms of energy such as heat,sound etc..
Eg: Collision between macroscopic objects

A collision in which the two particles move together after the collision is a perfectly inelastic collision.

Elastic Collisions in One Dimension

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision, or head-on collision.**



Before Collision After Collision

Consider two masses m_1 and m_2 making elastic collision in one dimension.
By the conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ --------------(1)}$$
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{----------------(2)}$$

By the conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \text{-----------(3)}$$
$$\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$
$$\frac{1}{2}m_1\,(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$$
$$m_1\,(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{-------------(4)}$$

Eqn $\frac{(4)}{(2)}$ ------------ $\frac{m_1\,(u_1^2-v_1^2)}{m_1(u_1-v_1)} = \frac{m_2(v_2^2-u_2^2)}{m_2(v_2-u_2)}$

$$\frac{(u_1+v_1)\,(u_1-v_1)}{(u_1-v_1)} = \frac{(v_2+u_2)(v_2-u_2)}{(v_2-u_2)}$$
$$u_1 + v_1 = v_2 + u_2 \quad \text{-------------(5)}$$
$$\mathbf{u_1 - u_2 = -(v_1 - v_2)} \text{--------(6)}$$

i.e., relative velocity before collision is numerically equal to relative velocity after collision.

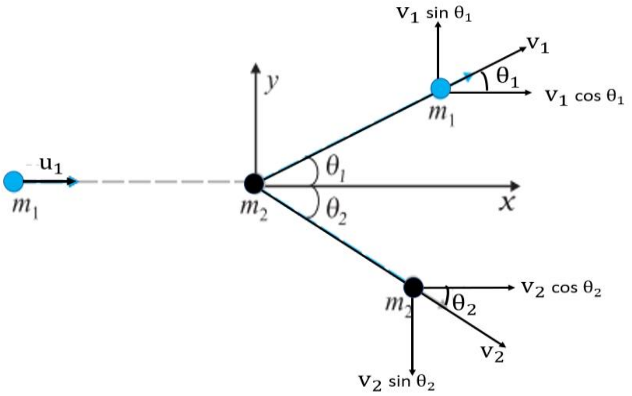
From eqn(5), $v_2 = u_1 + v_1 - u_2$

Substituting in eqn (1)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$
$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_1v_1 + m_2v_1$$
$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$
$$\mathbf{v_1 = \frac{(m_1-m_2)u_1}{m_1+m_2} + \frac{2m_2u_2}{m_1+m_2}} \text{ ------- (7)}$$

Similarly, $\mathbf{v_2 = \frac{(m_2-m_1)u_2}{m_1+m_2} + \frac{2m_1u_1}{m_1+m_2}}$ ------- (8)

Elastic Collisions in Two Dimensions



Consider the elastic collision of a moving mass m_1 with the stationary mass m_2.
Since momentum is a vector ,it has 2 equations in x and y directions.

Equation for conservation of momentum in x direction

$$\mathbf{m_1u_1 = m_1v_1cos\theta_1 + m_2v_2cos\theta_2}$$

Equation for conservation of momentum in y direction

$$0 = \mathbf{m_1v_1sin\theta_1 - m_2v_2sin\,\theta_2}$$

Equation for conservation of kinetic energy,(KE is a scalar quantity)

$$\frac{1}{2}\mathbf{m_1u_1^2} = \frac{1}{2}\mathbf{m_1v_1^2} + \frac{1}{2}\mathbf{m_2v_2^2}$$

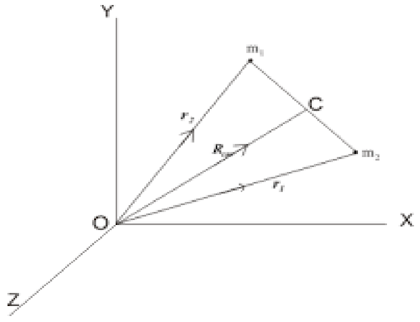
Chapter 7
Systems of Particles and Rotational Motion

Rigid Body

Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between different pairs of such a body do not change.

Centre Of Mass

The centre of is a hypothetical point where the entire mass of an object may be assumed to be concentrated to visualise its motion.



Consider a two particle system. Let C be the centre of mass which is at a distancev X from origin.

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M} \quad \text{where } M = m_1 + m_2$$

Motion of Centre of Mass

- **Position vector of centre of mass**

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots \ldots\ldots + m_n\vec{r}_n}{M} \quad \text{--------(1)}$$

where $M = m_1 + m_2 + \ldots\ldots + m_n$

- **Velocity of centre of mass**

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots\ldots + m_n\vec{v}_n}{M} \quad \text{--------------(2)}$$

- **Acceleration of centre of mass**

$$\vec{A} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots \ldots\ldots + m_n\vec{a}_n}{M} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

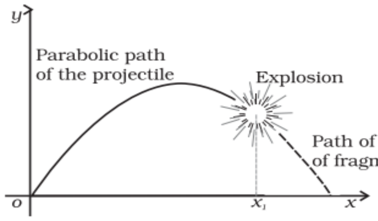
- **Force on centre of mass**

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots \ldots \ldots .. + m_n\vec{a}_n$$

$$\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \cdots \ldots \ldots .. + \vec{F}_n$$

$$\vec{F}_{ext} = M\vec{A}$$

The centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.



The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion.

- **Linear Momentum of centre of mass**

Velocity of centre of mass

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots\ldots + m_n\vec{v}_n}{M}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \ldots \ldots .. + m_n\vec{v}_n$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots \ldots \ldots .. + \vec{p}_n$$

Law of Conservation of Momentum for a System of Particles

If Newton's second law is extended to a system of particles,

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

When the sum of external forces acting on a system of particles is zero

$$\vec{F}_{ext} = 0$$

$$\frac{d\vec{P}}{dt} = 0$$

$$\vec{P} = \text{constant}$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the law of conservation of the total linear momentum of a system of particles.

$$\text{But } \vec{P} = M\vec{V}$$

$$M\vec{V} = \text{constant}$$

$$\vec{V} = constant$$

When the total external force on the system is zero the velocity of the centre of mass remains constant or the CM of the system is in uniform motion.

Vector Product or Cross product of Two Vectors

Vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

where A and B are magnitudes of $\vec{A}$ and $\vec{B}$

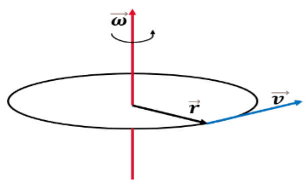
$\boldsymbol{\theta}$ is the angle between $\vec{A}$ and $\vec{B}$

$\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

The direction of $\vec{A} \times \vec{B}$ is given by right hand screw rule or right hand rule.

- $\hat{\imath} \times \hat{\imath} = 0$, $\hat{\jmath} \times \hat{\jmath} = 0$, $\hat{k} \times \hat{k} = 0$
- $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$
- $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath}$, $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

Angular Velocity and its Relation with Linear Velocity



The angular velocity is a vector quantity. $\vec{\omega}$ is directed along the fixed axis as shown.

The linear velocity of the particle is

$$\vec{v} = \vec{\omega} \times \vec{r}$$

It is perpendicular to both $\vec{\omega}$ and $\vec{r}$ and is directed along the tangent to the circle described by the particle.

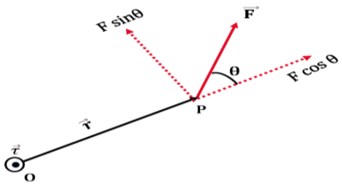
Angular acceleration

Angular acceleration $\vec{\alpha}$ is defined as the time rate of change of angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Torque or Moment of Force

The rotational analogue of force is torque or moment of force .



If a force $\vec{\mathbf{F}}$ acts on a single particle at a point P whose position with respect to the origin O is $\vec{r}$,then torque about origin o is

$$\vec{\tau} = \mathrm{r\,F\,sin\theta}$$
$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Torque has dimensions $M\,L^2T^{-2}$
- Torque is a vector quantity
- The SI unit of moment of force is Newton-metre (Nm)

Angular momentum of a particle

Angular momentum is the rotational analogue of linear momentum.

Angular momentum is a vector quantity. It could also be referred to as moment of (linear) momentum.

$$\vec{l} = \vec{r} \times \vec{p}$$
$$\vec{l} = rp\sin\theta$$

Relation connecting Torque and Angular momentum

$$\vec{l} = \vec{r} \times \vec{p}$$

Differentiating

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$
$$\frac{d\vec{l}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$
$$\vec{p} = m\vec{v} \quad , \frac{d\vec{r}}{dt} = \vec{v} \, , \frac{d\vec{p}}{dt} = \vec{F}$$
$$\frac{d\vec{l}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}$$
$$\vec{v} \times \vec{v} = 0 \, , \ (\vec{r} \times \vec{F} = \vec{\tau})$$
$$\frac{d\vec{l}}{dt} = 0 + \vec{\tau}$$
$$\vec{\tau} = \frac{d\vec{l}}{dt}$$

Thus, the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

This is the rotational analogue of the equation $\vec{F} = \frac{d\vec{p}}{dt}$, which expresses Newton's second law for the translational motion of a single particle.

Relation connecting Torque and Angular momentum for a system of particles

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

where $\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_n$

Law of Conservation of Angular momentum

For a system of particles

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

If external torque $\vec{\tau}_{ext} = 0$,

$$\frac{d\vec{L}}{dt} = 0$$
$$\vec{L} = \text{constant}$$

If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved i.e, remains constant.

Equilibrium of a Rigid Body

A rigid body is said to be in mechanical equilibrium, if it is in both translational equilibrium and rotational equilibrium.

i.e, for a body in mechanical equilibrium its linear momentum and angular momentum are not changing with time.

Translational Equilibrium

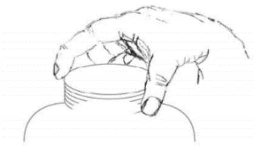
When the total external force on the rigid body is zero, then the total linear momentum of the body does not change with time and the body will be in translational equilibrium .

Rotational Equilibrium

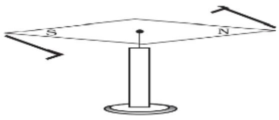
When the total external torque on the rigid body is zero, the total angular momentum of the body does not change with time and the body will be in rotational equilibrium .

Couple

A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation.

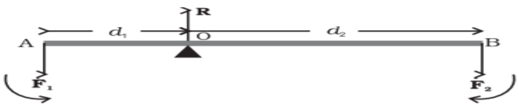


Our fingers apply a couple to turn the lid



The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple.

Principles of Moments



The lever is a system in mechanical equilibrium.

For rotational equilibrium the sum of moments must be zero,

$$\mathbf{d_1F_1 - d_2F_2 = 0}$$

The equation for the principle of moments for a lever is

$$\mathbf{d_1F_1 = d_2F_2}$$

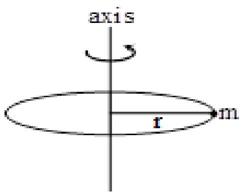
$$\textbf{load arm} \times \textbf{load} = \textbf{effort arm} \times \textbf{effort}$$

$$\text{Mechanical Advantage MA} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Moment of Inertia

Moment of Inertia is the rotational analogue of mass.

Moment of inertia is a measure of rotational inertia



The moment of inertia of a particle of mass m rotating about an axis is

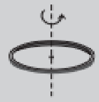
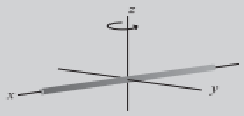
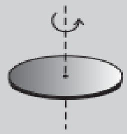
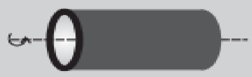
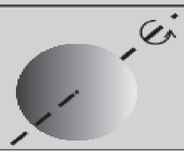
$$\mathbf{I = mr^2}$$

The moment of inertia of a rigid body is

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The moment of inertia of a rigid body depends on the mass of the body, its shape and size; distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

Moments of Inertia of some regular shaped bodies about specific axes

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius R	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$MR^2/2$
(5)	Circular disc, radius R	Diameter		$MR^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		MR^2
(7)	Solid cylinder, radius R	Axis of cylinder		$MR^2/2$
(8)	Solid sphere, radius R	Diameter		$2MR^2/5$

Rotational Kinetic energy

Cosider a particle of mass m rotating about an axis of radius r with angular velocity ω
The kinetic energy of motion of this particle is

$$kE = \frac{1}{2}mv^2$$

But $v = r\,\omega$

$$kE = \frac{1}{2}mr^2\omega^2$$

$I = mr^2$

$$\textbf{Rotational kE} = \frac{1}{2}\mathbf{I}\boldsymbol{\omega}^2$$

Radius of Gyration (k)

The radius of gyration can be defined as the distance of a mass point from the axis of roatation whose mass is equal to the whole mass of the body and whose moment of inertia is equal to moment of inertia of the whole body about the axis.

If K is the radius of gyration, we can write

$$I = Mk^2$$

$$k = \sqrt{\frac{I}{M}}$$

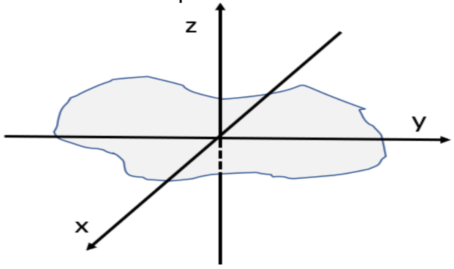
Theorems of Perpendicular and Parallel Axes

Perpendicular Axes Theorem

The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

Or

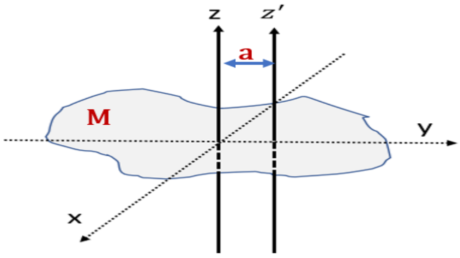
The moment of inertia of a plane lamina about z axis is equal to the sum of its moments of inertia about x-axis and y-axis, if the lamina lies in xy plane.



$$\mathbf{I_z = I_x + I_y}$$

(2) Parallel Axes Theorem

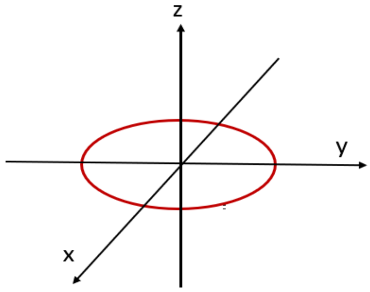
The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.



$$\mathbf{I_{z'} = I_z + Ma^2}$$

Applications Theorems of Momemt of Inertia

(1) Moment of Inertia of a Ring about One of its Diameter



By perpendicular axis theorem

$$I_z = I_x + I_y$$

But $I_x = I_y$

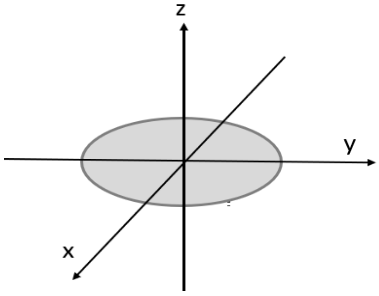
$$I_z = 2I_x$$

$$I_x = \frac{I_z}{2}$$

But $I_z = MR^2$

$$\mathbf{I_x = \frac{MR^2}{2}}$$

(2) Moment of Inertia of a Disc about One of its Diameter



By perpendicular axis theorem

$$I_z = I_x + I_y$$

But $I_x = I_y$

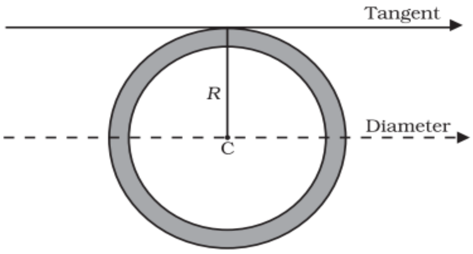
$$I_z = 2I_x$$

$$I_x = \frac{I_z}{2}$$

But $I_z = \frac{MR^2}{2}$

$$\mathbf{I_x = \frac{MR^2}{4}}$$

(3)Moment of inertia of a ring about a tangent to the circle of the ring



By parallel axis theorem

$$I_{z'} = I_z + Ma^2$$

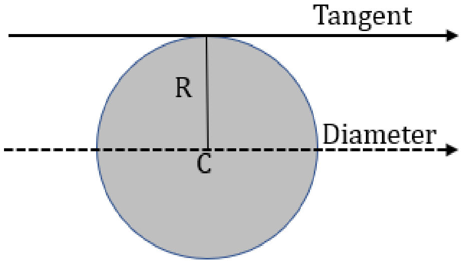
$$I_{tangent} = I_{\text{diameter}} + MR^2$$

But, $I_{\text{diameter}} = \frac{MR^2}{2}$

$$I_{tangent} = \frac{MR^2}{2} + MR^2$$

$$\mathbf{I_{tangent} = \frac{3}{2}MR^2}$$

(4)Moment of inertia of a disc about a tangent to the circle of the disc



By parallel axes theorem

$$I_{z'} = I_z + Ma^2$$

$$I_{tangent} = I_{diameter} + MR^2$$

$$\text{But, } I_{diameter} = \frac{MR^2}{4}$$

$$I_{tangent} = \frac{MR^2}{4} + MR^2$$

$$\mathbf{I_{tangent} = \frac{5}{4}MR^2}$$

Comparison of Translational and Rotational Motion

	Linear Motion	Rotational Motion about a Fixed Axis
1	Displacement x	Angular displacement θ
2	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4	Mass M	Moment of inertia I
5	Force $F = Ma$	Torque $\tau = I\,\alpha$
6	Work $dW = F\,ds$	Work $W = \tau\; d\theta$
7	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8	Power $P = F\,v$	Power $P = \tau\omega$
9	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

Conservation of angular momentum

If the external torque is zero, angular momentum is constant.

$$\vec{L} = \text{constant}$$

$$\text{But } \vec{L} = I\vec{\omega}$$

i.e., $I\vec{\omega} = \text{constant}$

When I increases ,ω decreases and vice versa, so that Iω is constant.

While the chair is rotating with considerable angular speed , if you stretch your arms horizontally, moment of inertia(I) increases and as a result, the angular speed(ω) is reduced.

If you bring back your arms closer to your body, moment of inertia(I) decreases and as a result, the angular speed(ω) increases again.

Kinetic Energy of Rolling Motion

Rolling motion is a combination of rotation and translation. The kinetic energy of a rolling body is the sum of kinetic energy of translation and kinetic energy of rotation.

Total KE = Translational KE + Rotational KE

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = mk^2$, where k = the radius of gyration

and v= R ω, $\omega = \frac{v}{R}$

$$K = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mk^2v^2}{R^2}$$

$$\mathbf{K = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)}$$

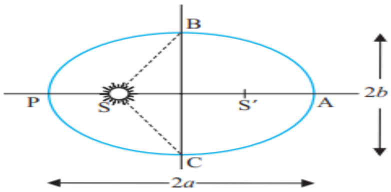
Here v is the velocity of centre of mass.

Chapter 8
Gravitation

Kepler's Laws

1.Law of orbits

All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.



PA is the major axis
BC is the minor axis

2.Law of areas

The line that joins any planet to the sun sweeps equal areas in equal intervals of time. i.e, areal velocity $\frac{\Delta\vec{A}}{\Delta t}$ is constant

The planets move slower when they are farther from the sun than when they are nearer.

The law of areas is a consequence of conservation of angular momentum.

3.Law of periods

The square of the time period of revolution of a planet is proportional to the cube of the semi- major axis of the ellipse traced out by the planet.

$$T^2 \propto a^3$$

Universal Law of Gravitation

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them .

$$F = G\frac{m_1m_2}{r^2}$$

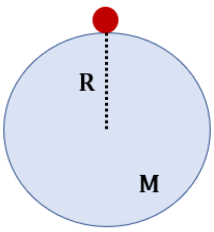
where G is the universal gravitational constant.

The Gravitational Constant

The value of the gravitational constant G was determined experimentally by English scientist Henry Cavendish in 1798. $G = 6.67\times10^{-11}\ N\ m^2/kg^2$

Acceleration due to gravity of the Earth

Consider a body of mass m on the surface of earth of mass M and radius R.



$$F = \frac{GMm}{R^2} \text{ -----------(1)}$$

By Newton's second law

$$F=mg \text{ -----------(2)}$$

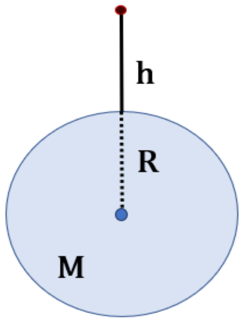
From Eq (1) and (2)

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

- **Acceleration due to gravity is independent of mass of the body.**
- The average value of g on the surface of earth is $9.8ms^{-2}$.

1.Acceleration due to gravity at a height h above the surface of the earth.



Acceleration due to gravity on the surface of earth

$$g = \frac{GM}{R^2} \text{------------}(1)$$

Acceleration due to gravity at a height above the surface of earth

$$g_h = \frac{GM}{(R+h)^2} \quad \text{----------}(2)$$

for , h << R, $g_h = \frac{GM}{R^2(1+\frac{h}{R})^2}$

$$g_h = \frac{GM}{R^2}(1+\frac{h}{R})^{-2}$$

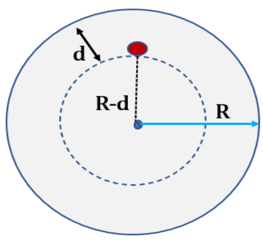
$$g_h = g(1+\frac{h}{R})^{-2}$$

Using binomial expression and neglecting higher order terms.

$$\mathbf{g_h \cong g\,(1-\frac{2h}{R})}$$

Thus, as we go above earth's surface, the acceleration due gravity decreases by a factor $(\mathbf{1-\frac{2h}{R}})$

2.Acceleration due to gravity at a depth d below the surface of the earth



Mass =volume x density

$$M = \frac{4}{3}\pi R^3\rho \text{ ------------}(1)$$

Acceleration due to gravity on the surface of earth

$$g = \frac{GM}{R^2} \quad \text{-------------}(2)$$

$$g = \frac{G}{R^2}(\frac{4}{3}\pi R^3\rho)$$

$$g = \frac{4}{3}\pi R\rho G \text{ ---------------}(3)$$

Acceleration due to gravity at a depth d below the surface of earth

$$g_d = \frac{4}{3}\pi(R-d)\rho G \text{ ---------}(4)$$

$$\frac{eq(4)}{eq(3)} \text{ ------ } \frac{g_d}{g} = \frac{\frac{4}{3}\pi(R-d)\rho G}{\frac{4}{3}\pi R\rho G}$$

$$\frac{g_d}{g} = \frac{(R-d)}{R}$$

$$\mathbf{g_d = g(1-\frac{d}{R})}$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor$(\mathbf{1-\frac{d}{R}})$

- The value of acceleration due to earth's gravity is maximum on its surface and decreases whether you go up or down.
- At the centre of earth acceleration due to earth's gravity is zero.

Example

At what height the value of acceleration due to gravity will be half of that on surface of earth. (Given the radius of earth R =6400km)

.
$$g_h = g(1+\frac{h}{R})^{-2}$$

$$g_h = \frac{g}{2}$$

$$\frac{g}{2} = g(1+\frac{h}{R})^{-2}$$

$$\frac{1}{2} = (1+\frac{h}{R})^{-2}$$

$$2 = (1+\frac{h}{R})^2$$

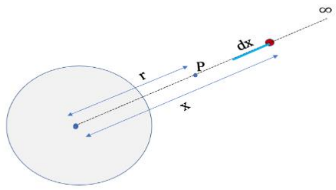
$$\sqrt{2} = 1+\frac{h}{R}$$

$$\frac{h}{R} = \sqrt{2}\text{-}1$$

$$h = (\sqrt{2}\text{-}1)\,R = (1.414\text{ -}1)\,6400 = 2650 \text{ km}$$

Gravitational Potential Energy

Gravitational potential energy at point is defined as the work done in displacing the particle from infinity to that point without acceleration.



The work done to give a displacement dx to the mass

$$dW = Fdx$$
$$dW = \frac{GMm}{x^2}dx$$

Total work done to move the mass from ∞ **to r**

$$W = \int_{\infty}^{r} \frac{GMm}{x^2}dx$$
$$W = \frac{-GMm}{r}$$

This work is is stored as gravitational PE in the body.

$$\mathbf{U = \frac{-GMm}{r}}$$

For unit mass m=1

So gravitational potential, $\mathbf{V = \frac{-GM}{r}}$

Escape speed

The minimum speed required for an object to reach infinity i.e. to escape from the earth's gravitational pull is called escape speed.

Let the body thrown from the surface of earth to infinity with velocity v_i.

$$\frac{1}{2}mv_i^2 - \frac{GMm}{r} = 0$$
$$\frac{1}{2}mv_i^2 = \frac{GMm}{R}$$
$$v_i^2 = \frac{2GM}{R}$$
$$\mathbf{v_e = \sqrt{\frac{2GM}{R}}}$$

- **Escape velocity is independent of mass of the body.**
- **Escape speed (or escape velocity) on the surface of earth is 11.2km/s**

Moon has no atmosphere. Why?

The escape speed of moon is about 2.3 km/s. which is less than the average speed of gas molecules of moon. Thus gas molecules escape from surface of moon and it has no atmosphere.

Earth Satellites

Earth satellites are objects which revolve around the earth.
Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.

- Satellites are of two types (1) Natural satellites and artificial satellites.
- Moon is the natural satellite of earth whose time period of revolution is 27.3 days.
- Artificial satellites are used for telecommunication, geophysics and meteorology etc.

Orbital Speed

The speed with which a satellites revolves around earth is called orbital speed.

$$F_{centripetal} = F_{gravitational}$$
$$\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$$
$$v^2 = \frac{GM}{(R+h)}$$
$$\boldsymbol{v_o = \sqrt{\frac{GM}{(R+h)}}}$$

If the satellite is very close to earth (R+h) ≈R , $\boldsymbol{v_o = \sqrt{\frac{GM}{R}}}$

Relation Connecting Escape Velocity and Orbital Velocity

Orbital Velocity ,$v_0 = \sqrt{\frac{GM}{R}}$

Escape Velocity ,$v_e = \sqrt{\frac{2GM}{R}}$

$$\mathbf{v_e = \sqrt{2}\, v_o}$$

Escape Velocity = $\sqrt{2}$ x Orbital Velocity

Period of a Satellite

Period of a satellite is the time required for a satellite to complete one revolution around the earth in a fixed orbit.

$$\text{Period T} = \frac{\text{circumference of the orbit}}{\textit{orbital speed}}$$

$$T = \frac{2\pi\,(R+h)}{\sqrt{\frac{GM}{(R+h)}}}$$

$$\mathbf{T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}}$$

Energy of an orbiting satellite

$$KE = \frac{1}{2}mv_o^2$$

$$v_o = \sqrt{\frac{GM}{R+h}}, \quad v_o^2 = \frac{GM}{R+h}$$

$$KE = \frac{1}{2} m\, x \frac{GM}{R+h}$$

$$KE = \frac{GMm}{2(R+h)}$$

$$PE = \frac{-GMm}{R+h}$$

$$\text{Energy} = KE + PE$$

$$E = \frac{GMm}{2(R+h)} + \frac{-GMm}{R+h}$$

$$\mathbf{E = \frac{-GMm}{2(R+h)}}$$

The total energy of an circularly orbiting satellite is negative, which means that the satellite is bound to the planet .If the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

Geostationary Satellites

Satellites in a circular orbits around the earth in the equatorial plane with period, T = 24 hours are called Geostationery Satellites .

- As the earth rotates with the same period, the satellite would appear fixed from any point on earth.
- Geostationary satellites are at a height of e 35800 km from the surface of earth.
- Geostationary satellites are widely used for telecommunications.
- Eg. The INSAT group of satellites sent up by India are Geostationary satellites.

Polar Satellites

Satellites which revolve around earth along poles in a north-south direction are called polar satellites.

- These are low altitude (h= 500 to 800 km) satellites.
- Since its time period is around 100 minutes it crosses any altitude many times a day.
- These satellites can view polar and equatorial regions at close distances with good resolution.
- Polar satellites are useful for remote sensing, meterology as well as for environmental studies of the earth.

Weightlessness

When an object is in free fall, it is weightless and this is called the phenomenon of weightlessness.

In a satellite around the earth, every part of the satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position. **Thus in the satellite everything inside it is in a state of free fall. Thus, in a manned satellite, people inside experience no gravity.**

Chapter 9
Mechanical Properties of Solids

A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required.

Elasticity

The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and such substances are called elastic .

Eg: Steel, Rubber

Steel is more elastic than rubber.

Plasticity

Some substances have no tendency to regain their previous shape on the removal of deforming force and they get permanently deformed. Such substances are called plastic and this property is called plasticity.

Eg:Putty and mud

Stress

The restoring force per unit area is known as stress.

If F is the force applied and A is the area of cross section of the body,

$$\text{Stress} = \frac{F}{A}$$

The SI unit of stress is N m^{-2} or pascal (Pa)

Dimensional formula of stress is [$ML^{-1}T^{-2}$]

Strain

Strain is defined as the **fractional change in dimension.**

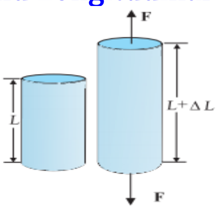
$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain has no unit and dimension.

There are three ways in which a solid may change its dimensions when an external force acts on it. As a result there are three types of stress and strain.

1. Longitudinal Stress and Longitudinal Strain
2. Shearing Stress and Shearing Strain
3. Hydraulic Stress and Hydraulic Strain (Volume Strain)

1.Longitudinal Stress and Longitudinal Strain



- **Longitudinal stress is defined as the restoring force per unit area when force is applied normal to the cross-sectional area of a cylinder.**

$$\text{Longitudinal stress} = \frac{F}{A}$$

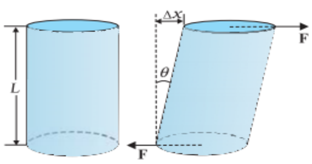
If the cylinder is stretched the stress is called **tensile stress** and If the cylinder is compressed it is called **compressive stress**.

- **Longitudinal strain is defined as the ratio of change in length(ΔL) to original length(L) of body.**

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

2.Shearing Stress and Shearing Strain



- **Shearing stress is defined as the restoring force per unit area when a tangential force is applied on the cylinder.**

$$\text{Shearing stress} = \frac{F}{A}$$

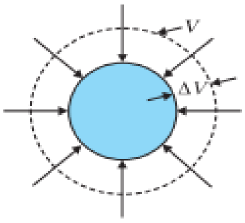
- **Shearing strain is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L**

$$\text{Shearing strain} = \frac{\Delta x}{L} = \tan\theta$$

Usually θ is very small, tan θ ≈ θ

$$\text{Shearing strain} = \theta$$

3.Hydraulic Stress and Hydraulic strain (Volume Strain)



When a solid sphere placed in the fluid, the force applied by the fluid acts in perpendicular direction at each point of the surface.

- **The hydraulic stress ie defined as the restoring force per unit area of solid sphere, placed in the fluid .**

$$\text{Hydraulic stress} = \frac{F}{A} = -P \text{ (pressure)}$$

The negative sign indicates that when pressure increases, the volume decreases.

- **Hydraulic strain (Volume strain))is defined as the ratio of change in volume (ΔV) to the original volume (V).**

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\text{Volume strain} = \frac{\Delta V}{V}$$

Hooke's Law

For small deformations the stress is directly proportional to strain. This is known as Hooke's law.

$$\text{Stress} \propto \text{Strain}$$

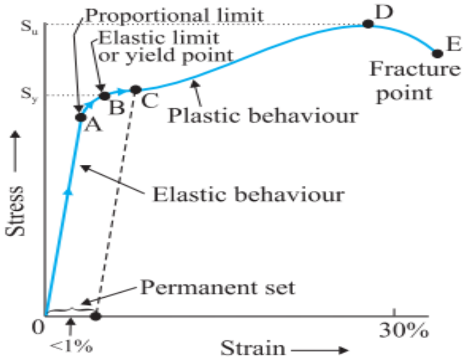
$$\text{stress} = k \times \text{strain}$$

$\frac{\text{Stress}}{\text{strain}} = k$ where k is a constant and is known as **Modulus of Elasticity.**

- The SI unit of modulus of elasticity is N m^{-2} or pascal (Pa)
  (same as that of stress,since strain is unitless)
- Dimensional formula is [$ML^{-1}T^{-2}$]

Stress-Strain Curve

A typical stress-strain curve for a metal is as shown in figure:



In the region from O to A

- The curve is linear. In this region, stress is proportional to strain .
- Hooke's law is obeyed.
- The point A on the curve is called **proportional limit.**

In the region from A to B

- Stress and strain are not proportional.
- Hooke's law is not obeyed.
- Nevertheless, the body is still elastic.
- **The point B in the curve is known as yield point or elastic limit.**
- **The stress corresponding to yield point is known as yield strength (S_y) of the material.**

In the region from B to D

- Beyond the point B ,the strain increases rapidly even for a small change in the stress. When the load is removed, at some point C between B and D, the body does not regain its original dimension.
- The material is said to have a permanent set. The material shows plastic behaviour in this region.
- **The point D on the graph is the ultimate tensile strength (S_u) of the material.**

In the region from D to E

- Beyond this point D, additional strain is produced even by a reduced applied force and fracture occurs at point E.
- The point E is called **Fracture Point.**
- If the ultimate strength and fracture points **D and E are close,** the material is said to be **brittle.**
- If **D and E are far apart,** the material is said to be **ductile.**

Elastomers

Substances like tissue of aorta, rubber etc. which can be stretched to cause large strains are called elastomers.

Elastic Moduli

The ratio of stress and strain, called modulus of elasticity. Depending upon the types of stress and strain there are three moduli of elasticity.

1. **Young's Modulus(Y)**
2. **Shear Modulus or Rigidity Modulus (G)**
3. **Bulk modulus(B)**

1.Young's Modulus(Y)

The ratio of longitudinal stress to longitudinal strain is defined as Young's modulus of the material .

$$Y=\frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y=\frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\mathbf{Y=\frac{FL}{A\,\Delta L}}$$

If $F=mg$ and $A=\pi r^2$

$$\mathbf{Y=\frac{mgL}{\pi r^2\,\Delta L}}$$

- SI unit of Young's modulus is N m^{-2} or Pa.
- For metals Young's moduli are large.
- Steel is more elastic than rubber as the Young's modulus of steel is large.
- Wood, bone, concrete and glass have rather small Young's moduli.

Why steel is preferred in heavy-duty machines and in structural designs?

Young's modulus of steel is greater than that of copper, brass and aluminium. It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy-duty machines and in structural designs.

2.Shear Modulus or Rigidity Modulus(G)

The ratio of shearing stress to the corresponding shearing strain is called the shear modulus or Rigidity modulus of the material .

$$G=\frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$G=\frac{\frac{F}{A}}{\frac{\Delta x}{L}}=\frac{\frac{F}{A}}{\theta}$$

$$\mathbf{G=\frac{F}{A\,\theta}}$$

- SI unit of shear modulus is N m^{-2} or Pa.
- Shear modulus is generally less than Young's modulus.
- For most materials G ≈ Y/3

3.Bulk Modulus(B)

The ratio of hydraulic stress to the corresponding hydraulic strain is called bulk modulus.

$$B=\frac{\text{Hydraulic stress}}{\text{Hydraulic strain}}$$

$$B=\frac{\frac{F}{A}}{\frac{\Delta V}{V}}=\frac{-P}{\frac{\Delta V}{V}}$$

$$\mathbf{B=\frac{-PV}{\Delta V}}$$

- SI unit of Bulk modulus is N m^{-2} or Pa.
- The negative sign indicates that when pressure increases, the volume decreases. That is, if p is positive, ΔV is negative.
- For a system in equilibrium, the value of bulk modulus B is always positive.

Compressibility(k)

The reciprocal of the bulk modulus is called compressibility.

$$k=\frac{1}{B}$$

$$\mathbf{k=\frac{-1}{P}\,\frac{\Delta V}{V}}$$

- The bulk moduli for solids are much larger than for liquids, which are again much larger than the bulk modulus for gases (air).
- Thus solids are least compressible whereas gases are most compressible.

Poisson's ratio

The ratio of lateral strain to longitudinal strain is called Poisson's ratio.

$$\text{Poisson's Ratio } \sigma=\frac{\text{Lateral Strain}}{\text{Longitudnal Strain}}$$

$$\sigma=\frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}}$$

$$\boldsymbol{\sigma=\frac{\Delta d}{\Delta L}\,\mathrm{x}\,\frac{L}{d}}$$

Poisson's ratio has no unit and dimension.

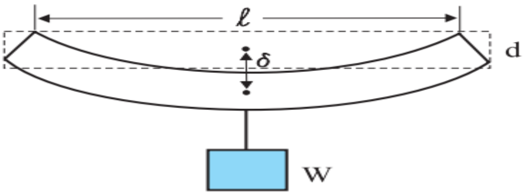
Applications of Elastic Behaviour of Materials

1.Cranes used for lifting and moving heavy loads have a thick metal rope . This is due to the fact that metals have greater youngs modulus.

Also, the elongation of the rope should not exceed the elastic limit. For this thicker rope of radius about 3 cm is recommended. A single wire of this radius would practically be a rigid rod. **So the ropes are always made of a number of thin wires braided together,** like in pigtails, for ease in manufacture, flexibility and strength.

2.The maximum height of a mountain on earth is ~10 km. The height is limited by the elastic properties of rocks.

3.In the construction of bridges and buildings , the beams should not bend too much or break. To reduce the bending for a given load, a material with a large Young's modulus Y is used.



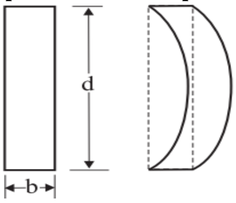
A beam of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by

$$\delta = \frac{W\,l^3}{4bd^3Y}$$

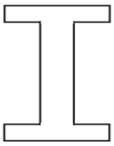
For a given load, the bending reduces when a material with a large Young's modulus Y is used.Bending can also be reduced by increasing thebreadth b, and depth d of the beam.

Buckling

Bending can be effectively reduced by increasing the depth d of the beam. But on increasing the depth, unless the load is exactly at the right place , the deep bar may bend sidewise(as in figure).This is called **buckling.**



To avoid buckling, beams with cross-sectional shape of I is used.



- This section provides a large load bearing surface and enough depth to prevent bending.
- This shape reduces the weight of the beam without sacrificing the strength .
- This shape reduces the cost.

Chapter 10
Mechanical Properties of Fluids

Liquids and gases can flow and are therefore, called fluids.
The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.

Basic difference between Liquids and Gases

A liquid is incompressible and has a free surface of its own. A gas is compressible and it expands to occupy all the space available to it. Gas has no free surface.

Pressure

The normal force(F) exerted by a fluid on an area A is called pressure.

Pressure, $P = \frac{F}{A}$

- Pressure is a scalar quantity.
- Its SI unit is Nm^{-2} or pascal (Pa)

A common unit of pressure is the atmosphere (atm). It is the pressure exerted by the atmosphere at sea level.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa.}$$

Density

Density ρ for a fluid of mass m occupying volume V is given by

$$\rho = \frac{m}{V}$$

- It is a positive scalar quantity.
- Its SI unit is kg m^{-3}.
- The dimensions of density are $[ML^{-3}]$.
- **The density of water at 4^0C (277 K) is 1000 kg m^{-3}.**
- A liquid is incompressible and its density is therefore, nearly constant at all pressures.
- Gases, on the other hand exhibit a large variation in densities with pressure.

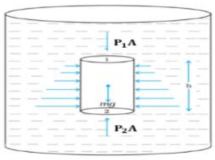
Relative Density

The relative density of a substance is the ratio of its density to the density of water at 4^0C.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4^0\text{C}}$$

It is a dimensionless positive scalar quantity.

Variation of Pressure with Depth



A fluid is at rest in a container. Consider a cylindrical element of fluid having area of base A and height h.
In equilibrium, the resultant vertical forces should be balanced.

$$P_2A = P_1A + mg$$
$$P_2A - P_1A = mg$$
$$(P_2 - P_1)A = mg$$

But $m = \rho V$, $V = hA$, $m = \rho hA$

$$(P_2 - P_1)A = \rho hA\,g$$
$$P_2 - P_1 = \rho\, gh$$

If the point 1 at the top of the fluid , which is open to the atmosphere, P_1 may be replaced by atmospheric pressure (P_a) and we replace P_2 by P

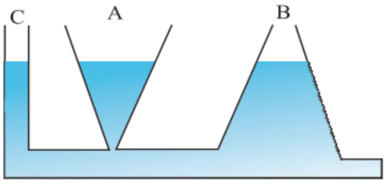
$$\text{Gauge pressure, } P - P_a = \rho\, gh$$

The excess of pressure, $P - P_a$, at depth h is called a gauge pressure at that point.

$$\text{Absolute Pressure, } P = P_a + \rho\, gh$$

Thus, the absolute pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount ρgh.

Hydrostatic paradox



The absolute pressure depends on the height of the fluid column and not on cross sectional or base area or the shape of the container. The liquid pressure is the same at all points at the same horizontal level (same depth). The result is appreciated through the example of **hydrostatic paradox.**

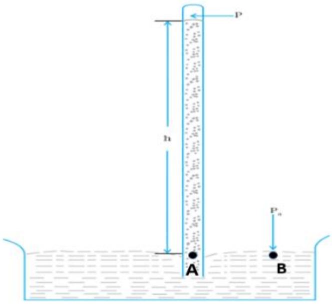
Atmospheric Pressure

It is the pressure exerted by the atmosphere at sea level.
The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Mercury barometer

Mercury barometer is used to measure Atmospheric Pressure. Italian scientist Evangelista Torricelli devised mercury barometer.



The pressure at point A =The pressure at point B.

Pressure at B = P_a (atmospheric pressure)
Pressure at A = ρgh

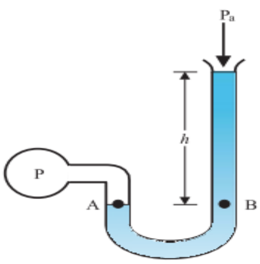
$$P_a = \rho gh$$

where ρ is the density of mercury and h is the height of the mercury column in the tube.

At sea level h= 76 cm and is equivalent to 1 atm.

Open-tube manometer

An open-tube manometer is a used for measuring Guage pressure or pressure differences.



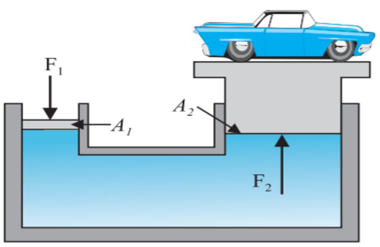
The pressure at A = pressure at point B
P= $P_a + \rho$ gh
$P - P_a = \rho gh$
The gauge pressure is proportional to manometer height h.

Pascal's law for transmission of fluid pressure

Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

Applications of Pascal's law

1.Hydraulic lift



The pressure on smaller piston

$$P = \frac{F_1}{A_1} \text{-------------}(1)$$

This pressure is transmitted equally to the larger cylinder with a larger piston of area A_2 producing an upward force F_2.

$$P = \frac{F_2}{A_2} \text{-------------}(2)$$

From eq(1) and (2) $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$\mathbf{F_2 = F_1 \frac{A_2}{A_1}}$$

Thus, the applied force has been increased by a factor of $\frac{A_2}{A_1}$ and this factor is the mechanical advantage of the device.

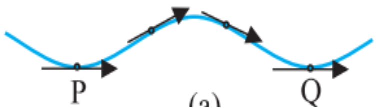
2.Hydraulic brakes

When we apply a force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area.
The pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.

Streamline Flow (Steady Flow)

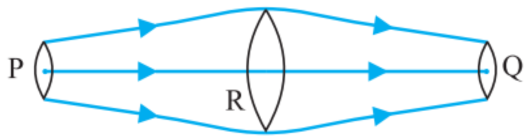
The study of the fluids in motion is known as fluid dynamics.
The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time.

- The velocity of a particular particle may change as it moves from one point to another.
- The path taken by a fluid particle under a steady flow is a streamline.
- **Streamline is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.**



- No two streamlines can cross, for if they do, an oncoming fluid particle can go either one way or the other and the flow would not be steady.

Equation of Continuity



The mass of liquid flowing out = The mass of liquid flowing in

$$\rho_P A_P v_P \Delta t = \rho_Q A_Q v_Q \Delta t = \rho_R A_R v_R \Delta t$$

If the fluid is incompressible $\rho_P = \rho_Q = \rho_R$

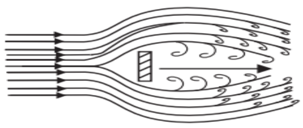
$$A_P v_P = A_Q v_Q = A_R v_R$$

$$Av = \text{constant}$$

This is called the equation of continuity and it is a statement of conservation of mass in flow of incompressible fluids.

Turbulent Flow

Steady flow is achieved at low flow speeds. Beyond a limiting value, called critical speed, the flow of fluid loses steadiness and becomes turbulent.



A jet of air striking a flat plate placed perpendicular to it is an example of turbulent flow.

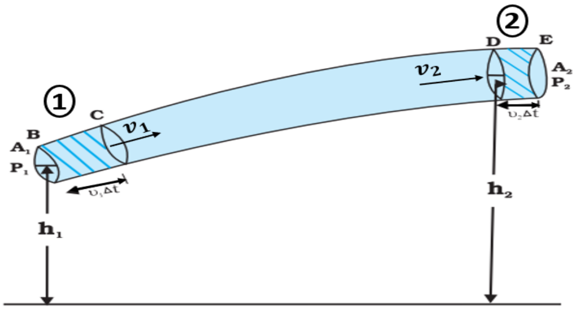
Bernoulli's Principle

Bernoulli's principle states that as we move along a streamline, the sum of the pressure , the kinetic energy per unit volume and the potential energy per unit volume remains a constant.

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

The equation is basically the conservation of energy **applied to non viscous fluid motion in steady state.**

Proof



The total work done on the fluid is

$$W_1 + W_2 = P_1\Delta V - P_2\Delta V$$

$$\mathbf{W_1 + W_2 = (P_1 - P_2)\Delta V}\text{-------------}(1)$$

The change in its kinetic energy is

$$\Delta K = \frac{1}{2}m\left(v_2^2 - v_1^2\right) \text{-------------}(2)$$

The change in gravitational potential energy is

$$\Delta U = mg(h_2 - h_1) \text{----------------}(3)$$

By work – energy theorem

$$W_1 + W_2 = \Delta K + \Delta U$$

$$(P_1 - P_2)\Delta V = \frac{1}{2}m\left(v_2^2 - v_1^2\right) + mg(h_2 - h_1)\text{-------------}(4)$$

Divide each term by ΔV to obtain ,

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1) \qquad \left(\rho = \frac{m}{\Delta V}\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}\text{----------}(5)$$

This is Bernoulli's theorem

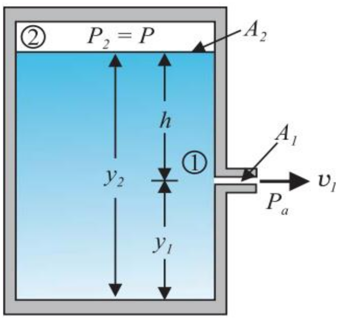
Note:-Bernoulli's theorem is applicable only to the streamline flow of non viscous and incompressible fluids.

Applications of Bernoulli's Principle

1.Speed of Efflux: Torricelli's Law

The word efflux means fluid outflow

Torricelli's law states that the speed of efflux of fluid through a small hole at a depth h of an open tank is equal to the speed of a freely falling body i.e, $\sqrt{2gh}$



Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom. According to Bernoulli principle

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Consider regions 1 and 2

According to equation of continuity ,since ($A_2 >> A_1$), $v_2 = 0$.

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

$$\frac{1}{2}\rho v_1^2 = \rho g(y_2 - y_1) + P - P_a$$

$$y_2 - y_1 = h$$

$$\frac{1}{2}\rho v_1^2 = \rho g h + P - P_a$$

$$v_1^2 = 2gh + \frac{2(P - P_a)}{\rho}$$

$$\mathbf{v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}}$$

If the tank is open to the atmosphere, then $P = P_a$

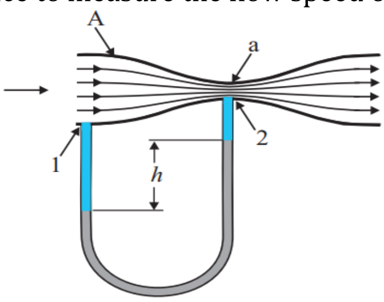
$$\mathbf{v_1 = \sqrt{2gh}}$$

This equation is known as Torricelli's law.

This is the speed of a freely falling body.

2.Venturi-meter

The Venturi-meter is a device to measure the flow speed of incompressible fluid.



According to Bernoulli principle

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

For points 1 and 2, $h_1 = h_2$

By equation of continuity , $Av_1 = av_2$, $v_2 = \frac{A}{a}v_1$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left(\frac{A}{a}v_1\right)^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A}{a}v_1\right)^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A}{a}\right)^2 - 1\right]$$

But $P_1 - P_2 = \rho_m g h$

h= height difference in manometer tube

ρ_m = density of liquid in manometer

$$\rho_m g h = \frac{1}{2}\rho v_1^2\left[\left(\frac{A}{a}\right)^2 - 1\right]$$

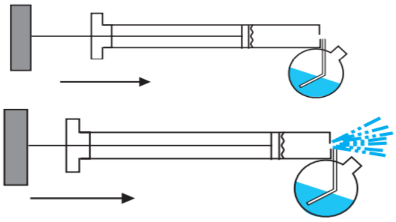
$$v_1^2 = \frac{\frac{2\rho_m g h}{\rho}}{\left[\left(\frac{A}{a}\right)^2 - 1\right]}$$

$$\mathbf{v_1 = \sqrt{\frac{2\rho_m g h}{\rho}}\left[\left(\frac{A}{a}\right)^2 - 1\right]^{-\frac{1}{2}}}$$

The speed of fluid at wide neck can be calculated using the above equation.

Venturimeter principle is used in,

- The carburetor of automobile has a Venturi channel (nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion.
- Filter pumps or aspirators
- Bunsen burner
- Atomisers
- Sprayers used for perfumes or to spray insecticides



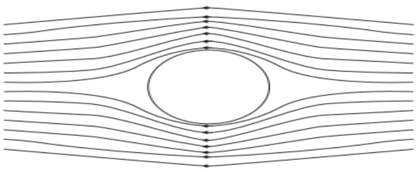
The spray gun. Piston forces air at high speeds causing a lowering of pressure at the neck of the container.

3.Blood Flow and Heart Attack

The artery may get constricted due to the accumulation of plaque on its inner walls. The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This may result in heart attack.

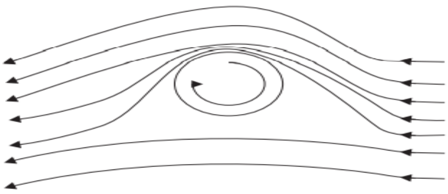
4.Dynamic Lift

(i)Ball moving without spin:



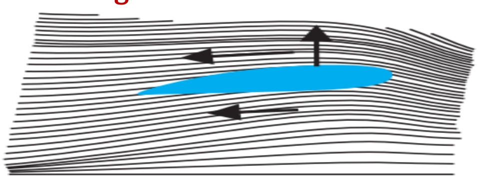
The velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(ii)Ball moving with spin:Magnus Effect



The ball is moving forward and relative to it the air is moving backwards. Therefore, the relative velocity of air above the ball is larger and below it is smaller. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spining is called **Magnus effect.**

(iii)Aerofoil or lift on aircraft wing

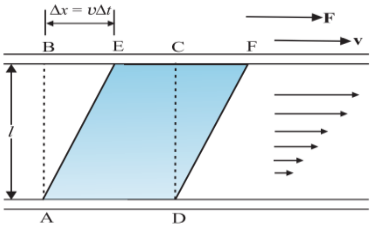


Aerofoil is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air. When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.

Viscosity

The internal frictional force that acts when there is relative motion between layers of the liquid is called viscosity.

Coefficient of viscosity(η)



The coefficient of viscosity(η)for a fluid is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{\text{Shearing stress}}{\text{Strain rate}} = \frac{\frac{F}{A}}{\frac{v}{l}}$$

$$\boldsymbol{\eta = \frac{Fl}{vA}}$$

- The SI unit of coefficient viscosity is poiseiulle (Pl).
- The dimensions are $[ML^{-1}T^{-1}]$
- The viscosity of liquids decreases with temperature while it increases in the case of gases.

Stokes' Law

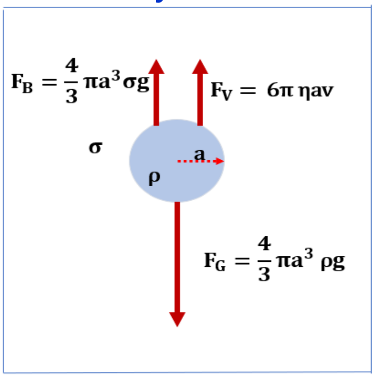
Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of coefficient of viscosity η is,

$$F = 6\pi\eta av$$

Terminal velocity

When an object falls through a viscous medium (raindrop in air), it accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally when viscous force plus buoyant force becomes equal to the force due to gravity(weight of the body), the net force and acceleration become zero. The sphere (raindrop) then descends with a constant velocity called terminal velocity.

Expression for Terminal velocity



Consider a raindrop in air. The forces acting on the drop are

1. Force due to gravity(weight,mg) acting downwards , $F_G = \frac{4}{3}\pi a^3 \rho g$
2. Buoyant force acting upwards, $F_B = \frac{4}{3}\pi a^3 \sigma g$
3. Viscous force, $F_V = 6\pi\eta av$

In equilibrium,

$$6\pi\eta av + \frac{4}{3}\pi a^3 \sigma g = \frac{4}{3}\pi a^3 g\rho$$

$$6\pi\eta av = \frac{4}{3}\pi a^3 (\rho - \sigma)g$$

Terminal velocity ,

$$v_t = \frac{2a^2(\rho - \sigma)g}{9\eta}$$

So the terminal velocity v_t depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

Reynolds Number

Osborne Reynolds defined a dimensionless number, whose value gives one an approximate idea whether the flow would be turbulent . This number is called the Reynolds number(R_e)

$$R_e = \frac{\rho v d}{\eta}$$

where ρ is the density of the fluid ,v is the speed of fluid, d stands for the dimension of the pipe, and η is the viscosity of the fluid.

R_e <1000 – The flow is streamline or laminar.
$R_e >$ 2000 – The flow is turbulent .
R_e between 1000 and 2000-The flow becomes unsteady .

- **The critical value of Reynolds number at which turbulence sets, is known as critical Reynolds number.** Turbulence dissipates kinetic energy usually in the fm of heat. Racing cars and planes are engineered to precision in order to minimise turbulence.
- Turbulence is sometimes desirable. The blades of a kitchen mixer induce turbulent flow and provide thick milk shakes as well as beat eggs into a uniform texture.

Surface Tension

The free surface of a liquid possess some additional energy and it behaves like a stretched elastic membrane. This phenomenon is known as surface tension. Surface tension is concerned with only liquid as gases do not have free surfaces.

Definition of Surface tension

Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance.

$$\text{Surface Tension, } S = \frac{\text{Force}}{\text{Length}}$$

- The SI Unit is Nm^{-1}
- Dimensional formula is MT^{-2}
- The value of surface tension depends on temperature.
- **The surface tension of a liquid decreases with temperature.**

Some effects of surface Tension

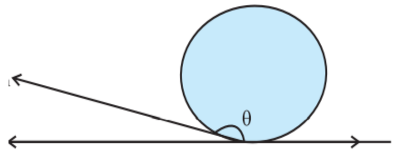
- Oil and water do not mix.
- Water wets you and me but not ducks.
- Mercury does not wet glass but water sticks to it.
- Oil rises up a cotton wick, inspite of gravity.
- Sap and water rise up to the top of the leaves of the tree.
- Hairs of a paint brush do not cling together when dry and even when dipped in water but form a fine tip when taken out of it.

Angle of Contact

The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as angle of contact(θ)

The value of θ determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

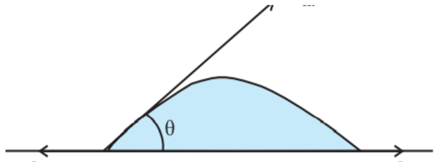
When Angle of contact is Obtuse:



When θ is an obtuse angle(greater than 90) then molecules of liquids are attracted strongly to themselves and weakly to those of solid, and liquid then does not wet the solid.

Eg: Water on a waxy or oily surface, Mercury on any surface.

When Angle of contact is Acute:



When θ is an acute angle (less than 90), the molecules of the liquid are strongly attracted to those of the solid and liquid then wets the solid.
Eg: Water on glass or on plastic, Kerosene oil on virtually anything .

Action Soaps and detergents

Soaps, detergents and dying substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective.

Action of Water proofing agents

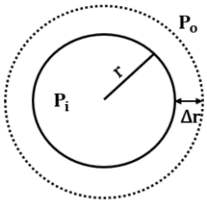
Water proofing agents are added to create a large angle of contact between the water and fibres.

Drops and Bubbles

Why are small drops and bubbles spherical?

Due to surface tension ,liquid surface has the tendency to reduce surface area. For a given volume sphere has minimum surface area. So small drops and bubbles are spherical.
For large drops the effect of gravity predominates that of surface tension and they get flattened.

Excess Pressure inside a spherical drop



Work done in expansion= Force x Displacement

= Excess pressure x Area x Displacement

$$\mathbf{W = (P_i - P_o) \times 4\pi r^2 \times \Delta r \text{ ------------------}(1)}$$

This workdone is equal to the increase in surface energy

Extra Surface energy = Surface tension x Increase in surface area

Extra surface energy = S x 8πrΔr ------------------(2)

The workdone = extra surface energy

$$\mathbf{(P_i - P_o) \times 4\pi r^2 \times \Delta r = 8\pi r \Delta r\, S \text{ --------------}(3)}$$

$$\mathbf{(P_i - P_o) = \frac{2S}{r}}$$

Excess Pressure Inside a Liquid Bubble

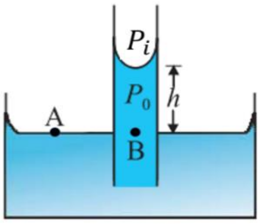
A bubble has two free surfaces.

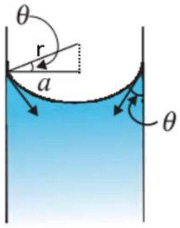
$$(P_i - P_o) = 2 \times \frac{2S}{r}$$

$$\mathbf{(P_i - P_o) = \frac{4S}{r}}$$

Capillary Rise

Due to the pressure difference across a curved liquid-air interface, water rises up in a narrow tube in spite of gravity. This is called capillary rise.





Consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water. The excess pressure on the concave meniscus

$$(P_i - P_o) = \frac{2S}{r}$$

$$\cos\theta = \frac{a}{r}\ ,\quad r = \frac{a}{\cos\theta}$$

$$(P_i - P_o) = \frac{2S}{\frac{a}{\cos\theta}}$$

$$(P_i - P_o) = \frac{2S\cos\theta}{a} \text{ ----------------(1)}$$

Consider two points A and B in the same horizontal level i.e, the points are at the same pressure.

$$\text{Pressure at A} = P_i$$

$$\text{Pressure at B} = P_o + h\rho g$$

$$P_i = P_o + h\rho g$$

$$P_i - P_o = h\rho g \text{ ----------------(2)}$$

From eq(1) and (2)

$$h\rho g = \frac{2S\cos\theta}{a}$$

$$h = \frac{2S\cos\theta}{\rho g a}$$

Thus capillary rise is a consequence of surface tension.
Capillary rise is larger, for capillary tube with smaller radius a.

Note:

If the liquid meniscus is convex, as for mercury, angle of contact θ will be obtuse . Then cos θ is negative and hence value of h will be negative. it is clear that the liquid will lower in the capillary and this is called **capillary fall or capillary depression.**

Detergents and Surface Tension

Detergents reduces the surface tension S (water-oil) and dirt can be removed by running water.

Chapter 11
Thermal Properties of Matter

Temperature and Heat

Temperature

Temperature is a relative measure, or indication of hotness or coldness.

- An object that has a higher temperature than another object is said to be hotter.
- SI unit of temperature is kelvin (K).
- ° C(degree celsius) , ° F(degree fahrenheit) are other commonly used unit of temperature.

Heat

Heat is the form of energy transferred between two systems or a system and its surroundings by virtue of temperature difference.

When the temperature of body and its surrounding medium are different, heat transfer takes place between the system and the surrounding medium, until the body and the surrounding medium are at the same temperature.

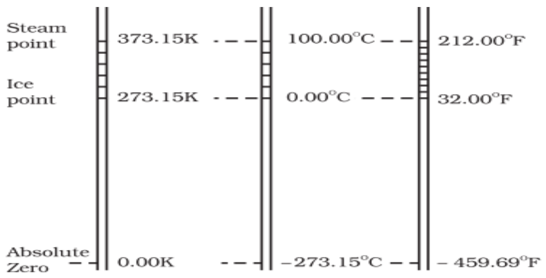
The SI unit of heat energy is joule (J)

Measurement of Temperature

A measure of temperature is obtained using a thermometer.

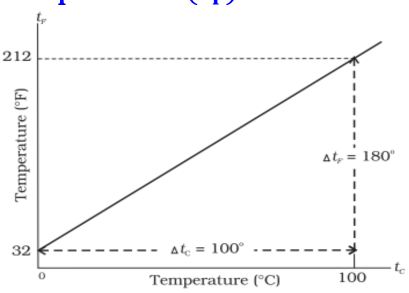
Variation of the volume of a liquid with temperature is used as the basis for constructing thermometers. Mercury and alcohol are the liquids used in most liquid-in-glass thermometers.

Comparison of the Kelvin, Celsius and Fahrenheit temperature scales.



- On Fahrenheit scale, there are 180 equal intervals between the ice and steam points.
- On Celsius scale, there are 100 equal intervals between the ice and steam points.
- On Kelvin scale, there are 100 equal intervals between the ice and steam points.

A plot of Fahrenheit temperature (t_F) versus Celsius temperature (t_C).



Temperature on Fahrenheit scale and Celsius scales are related by

$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

Temperature on Kelvin and Celsius scales are related by

$$T = t_C + 273.15$$

Ideal-Gas Equation and Absolute Temperature

Boyle's law

At constant temperature , the pressure of a quantity of gas is inversely proportional to volume.

$$P \propto \frac{1}{V}$$

$$PV = \text{constant.} \text{ ----------(1)}$$

Charles' law

At constant pressure , the volume of a quantity of gas is directly proportional to temperature .

$$V \propto T$$

$$\frac{V}{T} = \text{constant------------(2)}$$

Ideal gas law

Low density gases obey Boyle's law and Charles' law , which may be combined into a single relationship. Combining eq(1) and (2)

$$\frac{PV}{T} = \text{constant}$$

For any quantity of any dilute gas the law can be generalised as

$$\frac{PV}{T} = \mu R$$

$$PV = \mu RT$$

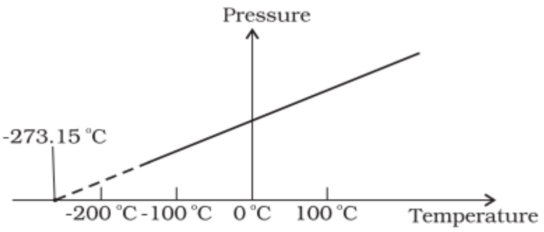
This is called ideal-gas equation

where, μ is the number of moles in the sample of gas.
R is called universal gas constant: $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

Pressure versus temperature curve of a low density gas kept at constant volume

For ideal gas, $PV = \mu RT$

If volume of a gas is kept constant, it gives $P \propto T$. A plot of pressure versus temperature gives a straight line in this case.



Absolute zero Temperature or Zero kelvin (0K)

The minimum temperature for an ideal gas is called **Absolute temperature or zero kelvin(0K).** This temperature is found to be **– 273.15 °C**

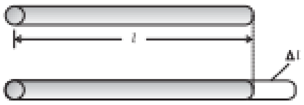
It is obtained by extrapolating the straight line of Pressure – temperature (at constant V) to the axis.

Thermal Expansion

The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion.
Three types of thermal expansions are
1.Linear expansion
2.Area expansion
3.Volume expansion

1.Linear Expansion

The expansion in length is called linear expansion.



If the substance is in the form of a long rod,
The fractional change in length, $\frac{\Delta l}{l} \propto \Delta T$.

$$\frac{\Delta l}{l} = \alpha_l \, \Delta T$$

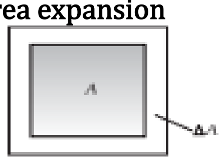
$$\alpha_l = \frac{\Delta l}{l\, \Delta T}$$

where α_l is known as the **coefficient of linear expansion** and is characteristic of the material of the rod.

- Metals expand more and have relatively high values of α_l .
- Copper expands about five times more than glass for the same rise in temperature.

2.Area Expansion

The expansion in area is called area expansion



The fractional change in area, $\frac{\Delta A}{A} \propto \Delta T$.

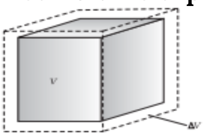
$$\frac{\Delta A}{A} = \alpha_a \Delta T$$

$$\boldsymbol{\alpha_a = \frac{\Delta A}{A\,\Delta T}}$$

where α_a is known as the **coefficient of area expansion .**

3. Volume Expansion

The expansion in volume is called volume expansion



The fractional change in volume, $\frac{\Delta V}{V} \propto \Delta T$

$$\frac{\Delta V}{V} = \alpha_v \Delta T$$

$$\boldsymbol{\alpha_v = \frac{\Delta V}{V\Delta T}}$$

where α_v is known as the **coefficient of volume expansion .**

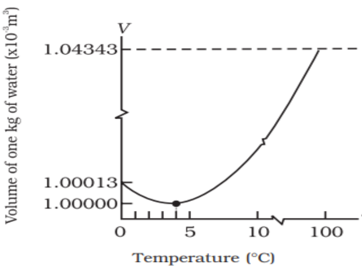
The value of α_v for alcohol (ethyl) is more than mercury and it expands more than mercury for the same rise in temperature.

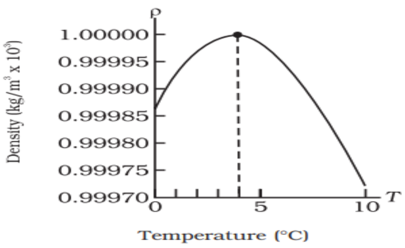
$$\alpha_a = 2\,\alpha_l \text{ ---------(1)}$$

$$\alpha_v = 3\,\alpha_l \text{ -----------(2)}$$

From eqs(1) and (2) $\alpha_l : \alpha_a : \alpha_v = 1 : 2 : 3$

Thermal Expansion of Water(Or) Anomalous Behavour of Water





Water exhibits an anomalous behaviour; it contracts on heating from 0 °C to 4 °C. When it is heated after 4 °C ,it expands like other liquids. This means that **water has minimum volume and hence maximum density at 4 °C .**

Why the bodies of water, such as lakes and ponds, freeze at the top first?

This is due to anomalous expansion of water. As a lake cools toward 4 °C, water near the surface becomes denser, and sinks. Then the warmer, less dense water near the bottom rises. When this layer cools below 4 °C, it freezes, and being less dense, remain at the surfaces. Thus water bodies freeze at the top first. Water at the bottom protects aquatic animal and plant life.

Thermal Stress

If the thermal expansion of a rod is prevented by fixing its ends rigidly, the rod acquires a compressive strain. The corresponding stress set up in the rod is called thermal stress.

Heat Capacity

Heat capacity (S) of a substance is the amount of heat required to raise the temperature of the substance by one unit.

$$\boldsymbol{S = \frac{\Delta Q}{\Delta T}}$$

Unit is JK^{-1}

Specific Heat capacity

Specific heat capacity (s) of a substance is the amount of heat required to raise the temperature of unit mass of the substance by one unit.

$$\text{Specific heat capacity} = \frac{\text{Heat capacity}}{\text{mass}}$$

$$s = \frac{S}{m}$$

$$s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Unit is $Jkg^{-1}K^{-1}$

$$\Delta Q = m\,s\,\Delta T$$

Molar Specific Heat Capacity

Molar Specific heat capacity (C) of a substance is the amount of heat required to raise the temperature of one mole of the substance by one unit.

$$C = \frac{S}{m}$$

$$C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

Unit is $Jmol^{-1}K^{-1}$

Specific Heat Capacities of Gases

As gas is compressible, heat transfer can be achieved by keeping either pressure or volume constant. So gases have two types of molar specific heat capacities.

Molar specific heat capacity at constant pressure C_p and

Molar specific heat capacity at constant volume C_v

Molar Specific Heat Capacity at Constant Pressure C_p

Molar specific heat capacity at constant pressure of a substance is the amount of heat required to raise the temperature of one mole of the substance by one unit keeping its pressure constant.

$$C_p = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)_p$$

Molar specific heat capacity at constant volume C_v

Molar specific heat capacity at constant volume of a substance is the amount of heat required to raise the temperature of one mole of the substance by one unit keeping its volume constant.

$$C_v = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)_v$$

Water has the highest specific heat capacity compared to other substances.

Specific heat capacity of water is 4186 is $Jkg^{-1}K^{-1}$

- For this reason water is used as a coolant in automobile radiators as well as a heater in hot water bags.
- Owing to its high specific heat capacity, the water warms up much more slowly than the land during summer and consequently wind from the sea has a cooling effect.
- In desert areas, the earth surface warms up quickly during the day and cools quickly at night.

Calorimetry

Calorimetry means measurement of heat.

Calorimeter

A device in which heat measurement can be made is called a calorimeter.

Change of State

Matter normally exists in three states: solid, liquid, and gas.

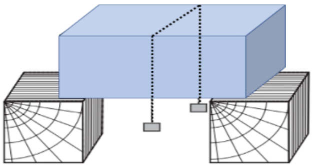
A transition from one of these states to another is called a change of state. The temperature of the system does not change during change of state.

Change of state from solid to liquid

The change of state from solid to liquid is called melting and from liquid to solid is called fusion.

- Both the solid and liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid.
- **The temperature at which the solid and the liquid states of the substance in thermal equilibrium with each other is called its melting point.**
- **Melting point decrease with increase in pressure** . The melting point of a substance at standard atomspheric pressure is called its normal melting point

Regelation



When the wire passes through the ice slab , ice melts at lower temperature due to increase in pressure. When the wire has passed, water above the wire freezes again. This phenomenon of refreezing is called **regelation.**

Skating is possible on snow due to the formation of water below the skates. Water is formed due to the increase of pressure and it acts as a lubricant.

The change of state from liquid to vapour

The change of state from liquid to vapour (or gas) is called vaporisation and from vapour to liquid is called condensation.

- The temperature remains constant until the entire amount of the liquid is converted into vapour
- **The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point.**
- **The boiling point increases with increase in pressure and decreases with decreases in pressure.** The boiling point of a substance at standard atmospheric pressure is called its normal boiling point.
  - Cooking is difficult on hills. At high altitudes, atmospheric pressure is lower, boiling point of water decreases as compared to that at sea level.
  - Boiling point is increased inside a pressure cooker by increasing the pressure. Hence cooking is faster.

Sublimation

The change from solid state to vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime.

Eg: Dry ice (solid CO2) , Iodine, Camphor

During the sublimation process both the solid and vapour states of a substance coexist in thermal equilibrium.

Change of state	
Solid to Liquid	Melting
Liquid to Solid	Fusion
Liquid to Gas	Vaporisation
Gas to Liquid	Condensation
Solid to Gas	Sublimation

Latent Heat

The amount of heat per unit mass transferred during change of state of the substance is called latent heat of the substance for the process.

The heat required during a change of state depends upon the heat of transformation and the mass of the substance undergoing a change of state.

$$Q = mL$$

$$L = \frac{Q}{m}$$

where L is known as latent heat and is a characteristic of the substance.

SI unit of Latent Heat is J kg^{-1}

The value of L also depends on the pressure. Its value is usually quoted at standard atmospheric pressure

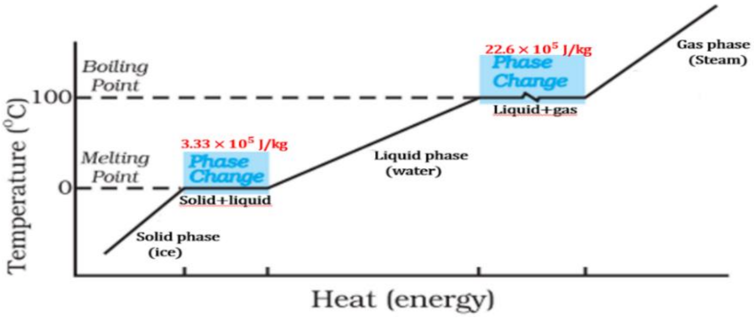
Latent Heat of Fusion (L_f)

The latent heat for a solid -liquid state change is called the latent heat of fusion (L_f) or simply heat of fusion.

Latent Heat of Vaporisation (L_v)

The latent heat for a liquid-gas state change is called the latent heat of vaporisation (L_v) or heat of vaporisation.

Temperature versus heat for water at 1 atm pressure (not to scale).



The slopes of the phase lines are not same, which indicate that specific heats of the various states are not equal. When slope of graph is less, it indicates a high specific heat capacity .

- The specific heat capacity of water is greater than that of ice.

  $$\Delta Q = m\, s\, \Delta T$$

  The amount of heat required ,ΔQ in liquid phase will be greater than that in solid phase for same ΔT.
  So slope of liquid phase is less than that of solid phase.
- For water, the latent heat of fusion is $\mathbf{L_f} = 3.33 \times 10^5\ \mathrm{J\,kg^{-1}}$.
  That is 3.33×10^5 J of heat are needed to melt 1 kg of ice at 0 °C.
  For water, the latent heat of vaporisation is $\mathbf{L_v} = 22.6 \times 105\ \mathrm{J\,kg^{-1}}$.
  That is 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100 °C.

Why burns from steam are usually more serious than those from boiling water?

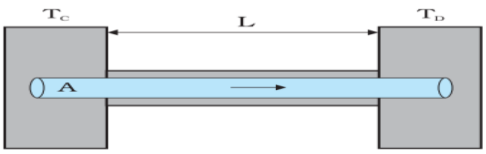
For water, the latent heat of vaporisation is $\mathbf{L_v} = 22.6 \times 10^5\ \mathrm{J\,kg^{-1}}$.
That is 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100 °C. So, steam at 100 °C carries $22.6 \times 10^5\ \mathrm{J\,kg^{-1}}$ more heat than water at 100 °C. This is why burns from steam are usually more serious than those from boiling water.

Heat Transfer

There are three distinct modes of heat transfer :
conduction, convection and radiation

1.Conduction

Conduction is the mechanism of transfer of heat between two adjacent parts of a body because of their temperature difference.



$$H = K\,A\,\frac{T_C - T_D}{L}$$

The constant of proportionality K is called the **thermal conductivity** of the material. The greater the value of K for a material, the more rapidly will it conduct heat. The SI unit of K is $\mathrm{Js^{-1}m^{-1}K^{-1}}$ or $\mathrm{Wm^{-1}K^{-1}}$

- Gases are poor thermal conductors while liquids have conductivities intermediate between solids and gases.
- Metals are good thermal conductors.
- Wood , glass and wool have small thermal conductivities.
- Some cooking pots have copper coating on the bottom. Being a good conductor of heat, copper promotes the distribution of heat over the bottom of a pot for uniform cooking.
- Plastic foams, on the other hand, are good insulators, mainly because they contain pockets of air.

- Houses made of concrete roofs get very hot during summer days, because thermal conductivity of concrete is moderately high. Therefore, people usually prefer to give a layer of earth or foam insulation on the ceiling so that heat transfer is prohibited and keeps the room cooler.

2.Convection

Convection is a mode of heat transfer by actual motion of matter. It is possible only in fluids.

Convection can be natural or forced.

Natural Convection

In natural convection, gravity plays an important part. When a fluid is heated from below, the hot part expands and, therefore, becomes less dense. Because of buoyancy, it rises and the upper colder part replaces it. This again gets heated, rises up and is replaced by the colder part of the fluid.

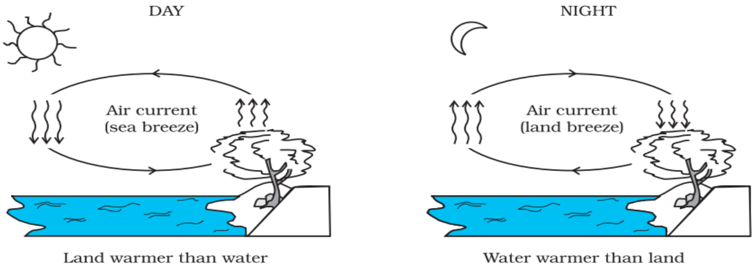
Eg: Sea breeze, Land breeze, Trade wind

1.Sea breeze

During the day, the ground heats up more quickly than large water bodies. This is due to greater specific heat capacity of water. The air in contact with the warm ground is heated . It expands, becomes less dense and rises . Then cold air above sea moves to fill this space and is called as sea breeze .

2.Land breeze

At night, the ground loses its heat more quickly, and the water surface is warmer than the land. The air in contact with water is heated. It expands, becomes less dense and rises . Then cold air above the ground moves to fill this space and is called as land breeze .



3.Trade wind

The surface of the earth at the equator is heated more by sun rays than poles. The hot air at equator expands, becomes less dense and rises . Then cold air from poles moves to the equator. This is called trade wind.

Forced Convection

In forced convection, material is forced to move by a pump or by some other physical means.

Eg: Forced-air heating systems in home
The human circulatory system
The cooling system of an automobile engine.

In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection and maintaining it at a uniform temperature.

3.Radiation

The mechanism for heat transfer which does not require a medium is called radiation.

The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation. The energy so radiated by electromagnetic waves is called radiant energy. All bodies emit radiant energy, whether they are solid, liquid or gases.

Heat is transferred to the earth from the sun through empty space as radiation.

Greenhouse Effect

The earth's surface radiates the energy absorbed from the sun.The wavelength of these radiation lies in the Infrared region. A large portion of these radiation is absorbed by greenhouse gases, namely, carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), chloroflurocarbon(CF_xCl_x)and tropospheric ozone (O_3). This heats up the atmosphere and it gives more energy to earth. This cycle continues until no radiation is available for absorption. The net result is the heating up of earth's surface and atmosphere. This is known as Greenhouse Effect.

Without the Greenhouse Effect, the earth's temperature would have been -18^0C.

Concentration of greenhouse gases is increased due to human activities, making the earth warmer . Because of global warming ice caps are melting faster, sea level is rising and weather is changing. This may cause problem for human life,plants and animals .

Newton's Law of Cooling

Newton's Law of Cooling says that the rate of loss of heat(rate of cooling) of a body is proportional the difference of temperature of the body and the surroundings.

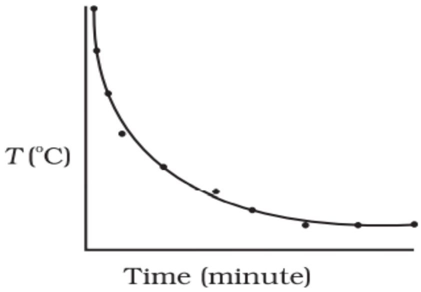
$$-\frac{dQ}{dT} = k(T_2 - T_1)$$

Where T_1 is the temperature of the surrounding medium

T_2 is the temperature of the body

k is a positive constant depending upon the area and nature of the surface of the body

Curve showing cooling of hot water with time



Chapter 12
Thermodynamics

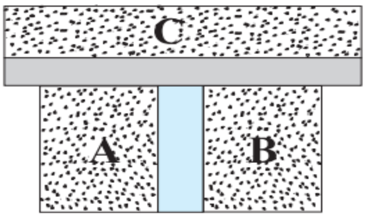
Thermodynamics

Thermodynamics is a branch of physics which deals with the study of heat, temperature and their inter conversion of heat energy into other forms of energy.

Zeroth Law of Thermodynamics

R.H. Fowler formulated this law in 1931 long after the first and second Laws of thermodynamics were stated and so numbered.

Zeroth Law of Thermodynamics states that 'two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other'.



Systems A and B are separated by an adiabatic wall, while each is in contact with a third system C via a conducting wall. In this case A will be in thermal equilibrium with B.

Thus if A and B are separately in equilibrium with C,

$T_A = T_C$ and $T_B = T_C$.

This implies that $T_A = T_B$

i.e. the systems A and B are also in thermal equilibrium.

i. e, If $T_A = T_C$ and $T_B = T_C$ then $T_A = T_B$

First Law of Thermodynamics

The heat supplied supplied to the system is partly used to increase the internal energy of the system and the rest is used to do work on the environment .

$$\Delta Q = \Delta U + \Delta W$$

ΔQ = Heat supplied to the system by the surroundings
ΔW = Work done by the system on the surroundings
ΔU = Change in internal energy of the system

Specific Heat Capacities of Gases

As gas is compressible, heat transfer can be achieved by keeping either pressure or volume constant. So gases have two types of molar specific heat capacities.

Molar specific heat capacity at constant pressure C_p

$$C_p = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)_p$$

Molar specific heat capacity at constant volume C_v

$$C_v = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)_v$$

Relation connecting C_p and C_v – Mayer's relation

Molar specific heat capacity at constant volume,

$$C_v = \left(\frac{\Delta Q}{\Delta T}\right)_v \qquad \text{(for 1 mole)}$$

$\Delta Q = \Delta U + P\,\Delta V$
At constant volume, $\Delta V = 0$,
$\Delta Q = \Delta U$

$$C_v = \left(\frac{\Delta U}{\Delta T}\right)_v$$

$$C_v = \frac{\Delta U}{\Delta T} \text{---------(1)}$$

Molar specific heat capacity at constant pressure

$$C_p = \left(\frac{\Delta Q}{\Delta T}\right)_p$$

At constant pressure, $\Delta Q = \Delta U + P\,\Delta V$

$$C_p = \left(\frac{\Delta U}{\Delta T}\right)_p + \left(P\frac{\Delta V}{\Delta T}\right)_p$$

$$\mathbf{C_p = \frac{\Delta U}{\Delta T} + \left(P\frac{\Delta V}{\Delta T}\right)_p \text{ -----------(2)}}$$

$PV = RT$

$P\left(\frac{\Delta V}{\Delta T}\right)_p = R$

$$C_p = \frac{\Delta U}{\Delta T} + R$$

Substituting from eq(1) $\quad C_p = C_v + R$

$$\mathbf{C_p - C_v = R}$$

This is called Mayer's relation.

C_p is always greater than C_v . Why

When gas is heated at constant volume, the entire heat is used to increase the internal energy of the gas. But when the gas is heated at constant pressure, the heat is used to increase the internal energy and also to do external work during expansion. $\frac{\Delta U}{\Delta T}$ is the same in both cases. Hence $\mathbf{C_p}$ is greater than $\mathbf{C_v}$.

Thermodynamic State Variables and Equation of State

Every equilibrium state of a thermodynamic system is completely described by specific values of some macroscopic variables, also called **state variables**.

For example, an equilibrium state of a gas is completely specified by the values of pressure, volume, temperature, and mass (and composition if there is a mixture of gases).

Equation of state

The connection between the state variables is called the equation of state.

Eg: For an ideal gas, the equation of state is the ideal gas relation

$$\mathbf{P\,V = \mu\,R\,T}$$

Extensive and Intensive Variables

The thermodynamic state variables are of two kinds:
Extensive and Intensive.

Extensive Variables

Extensive variables indicate the 'size' of the system.
(If we imagine ,to divide a system in equilibrium into two equal parts, the variables whose values get halved in each part are extensive.)

Eg:Internal energy, Volume , Mass

Intensive Variables

Intensive variables do not indicate the 'size' of the system.
(If we imagine, to divide a system in equilibrium into two equal parts, the variables that remain unchanged for each part are intensive.)

Eg: Pressure, Temperature , Density

Quasi-static process

The name quasi-static means nearly static.

A quasi-static process is an infinitely slow process such that the system remains in thermal and mechanical equilibrium with the surroundings throughout.

In a quasi-static process, the pressure and temperature of the environment can differ from those of the system only infinitesimally.

Eg: Processes that are sufficiently slow and do not involve accelerated motion of the piston, large temperature gradient, etc. are reasonably approximation to an ideal quasi-static process.

Some special thermodynamic processes

Type of processes	Feature
Isothermal	Temperature constant
Isobaric	Pressure constant
Isochoric	Volume constant
Adiabatic	No heat flow between the system and the surroundings ($\Delta Q = 0$)

1.Isothermal process.

A process in which the temperature of the system is kept fixed throughout is called an isothermal process.

For isothermal process $T = constant$.

So internal energy does not change, $\Delta U=0$

- Eg: Change of state (Melting, fusion, vaporistion..)
- The expansion of a gas in a metallic cylinder placed in a large reservoir of fixed temperature is an example of an isothermal process.

Equation of state for an isothermal process

For an ideal gas, $PV = \mu RT$

If an ideal gas goes isothermally from its initial state to the final state , its temperature remains constant

$$PV = constant$$

This is the equation of state for an isothermal process.

Work done by an ideal gas during an isothermal process

Consider an ideal gas undergoes a change in its state isothermally (at temperature T) from (P_1 ,V_1) to the final state (P_2 , V_2).

$$W = \int_{V_1}^{V_2} P\, dV$$

$$PV = \mu RT \quad \text{(for 1 mole)}$$

$$P = \frac{\mu RT}{V}$$

$$W = \int_{V_1}^{V_2} \frac{\mu RT}{V}\, dV$$

$$W = \mu RT \int_{V_1}^{V_2} \frac{1}{V}\, dV$$

$$W = \mu RT\, [\ln V]_{V_1}^{V_2}$$

$$W = \mu RT\, [\ln V_2 - \ln V_1]$$

$$W = \mu RT \ln \left[\frac{V_2}{V_1}\right]$$

Isothermal expansion,

For Isothermal expansion, $V_2 > V_1$ and hence $W > 0$ (workdone is positive)

That is, in an isothermal expansion, the gas absorbs heat and work is done by the gas on the environment.

Isothermal compression

In isothermal compression $V_2 < V_1$ and hence $W < 0$ (workdone is negative)

That is, In an isothermal compression, work is done on the gas by the environment and heat is released.

2.Adiabatic process

In an adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero.

$$\Delta Q = 0$$

Equation of state for an adiabatic process

$$PV^{\gamma} = \text{constant} \quad \text{Or} \quad T\,V^{\gamma-1} = \text{constant}$$

where γ is the ratio of specific heats (ordinary or molar) at constant pressure and at constant volume.

$$\gamma = \frac{C_p}{C_v}$$

Workdone by an Ideal gas during an Adiabatic Process

$$W = \int_{V_1}^{V_2} P\,dV$$

$$PV^{\gamma} = k \quad , \quad P = \frac{k}{V^{\gamma}}, \quad P = k\,V^{-\gamma}$$

$$W = k\int_{V_1}^{V_2} V^{-\gamma}\,dV$$

$$W = k\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$W = \frac{k}{1-\gamma}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right]$$

$$W = \frac{1}{1-\gamma}\left[\frac{k}{V_2^{\gamma-1}} - \frac{k}{V_1^{\gamma-1}}\right]$$

$$PV^{\gamma} = k$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = k$$

$$W = \frac{1}{1-\gamma}\left[\frac{P_2V_2^{\gamma}}{V_2^{\gamma-1}} - \frac{P_1V_1^{\gamma}}{V_1^{\gamma-1}}\right]$$

$$W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right] \text{--------------(1)}$$

$$PV = \mu R T$$

$$W = \frac{1}{1-\gamma}\left[\mu R T_2 - \mu R T_1\right]$$

$$W = \frac{\mu R}{1-\gamma}\left[T_2 - T_1\right] \text{--------------------(2)}$$

Or

$$W = \frac{\mu R}{\gamma-1}\left[T_1 - T_2\right] \text{--------------------(3)}$$

Adiabatic expansion

In adiabatic expansion, the work is done by the gas ($W > 0$), we get $T_2 < T_1$ i.e., the temperature of the gas lowers.

Adiabatic compression

In Adiabatic compression, work is done on the gas ($W < 0$), we get $T_2 > T_1$. i.e., the temperature of the gas rises.

3.Isochoric process

In an isochoric process, V is constant.

Workdone in an isochoric process

$$\Delta W = P\,\Delta V$$

For isochoric process, $\Delta V = 0$

$$\Delta W = 0$$

In an isochoric process no work is done on or by the gas.

4.Isobaric Process

In an isobaric process, P is constant.

Work done by the gas in an Isobaric process

Work done by the gas is

$$\Delta W = P\,\Delta V$$

$$W = P\,(V_2 - V_1)$$

or

$$W = \mu R\,(T_2 - T_1)$$

Cyclic Process

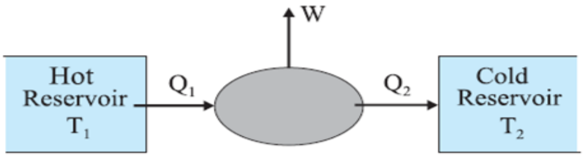
In a cyclic process, the system returns to its initial state.
Since internal energy is a state variable, $\Delta U = 0$ for a cyclic process

Heat Engines

Heat engines convert heat energy into mechanical energy.
Heat engine is a device by which a system is made to undergo a cyclic process that results in conversion of heat to work.

Heat engines consists of :

(1) Working substance–the system.
Eg:A mixture of fuel vapour and air in a gasoline or diesel engine or steam in a steam engine are the working substances.
(2) An external reservoir at some high temperature T_1 called source
(3) An external reservoir at some lower temperature T_2 called sink.



The working substance absorbs a total amount of heat Q_1 from the source at higher temperature, some external work is done by it on the environment and releases remaining amount of heat Q_2 to the sink at lower temperature T_2.

$$Q_1 = W + Q_2$$

$$W = Q_1 - Q_2$$

Efficiency of heat engine(η)

The efficiency (η) of a heat engine is defined by

$$\eta = \frac{W}{Q_1}$$

where Q_1 is the heat absorbed by the system from the source in one complete cycle
W is the work done by the system on the environment in a cycle.

$$W = Q_1 - Q_2$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

For $Q_2 = 0$, $\eta = 1$,

i.e., the engine will have 100% efficiency in converting heat into work.
Such an ideal engine with $\eta = 1$ (100% efficiency) is never possible, even if we can eliminate various kinds of losses associated with actual heat engines.

External and Internal combustion engines

The mechanism of conversion of heat into work varies for different heat engines. Basically, there are two types of heat engines: External combustion engines and Internal combustion engines

External Combustion Engine

In an external combustion engine, the system is heated by an external furnace.
Eg: steam engine

Internal Combustion Engines

In an internal combustion engine, the system is heated internally by an exothermic chemical reaction.

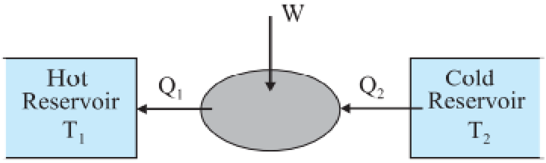
Eg: Petrol engine ,Diesel engine.

Refrigerators and Heat Pumps

A heat pump is the same as a refrigerator. What term we use depends on the purpose of the device. If the purpose is to cool a portion of space, like the inside of a chamber, and higher temperature reservoir is surrounding, we call the device a refrigerator; if the idea is to pump heat into a portion of space (the room in a building) when the outside environment is cold, the device is called a heat pump.
Both are reverse of a heat engine.

Refrigerator

A refrigerator is the reverse of a heat engine.
The purpose of a refrigerator is to cool a portion of space, like the inside of a chamber, and higher temperature reservoir is surrounding.



The Coefficient of Performance

The coefficient of performance (α) of a refrigerator is given by

$$\alpha = \frac{Q_2}{W}$$

where Q_2 is the heat extracted from the cold reservoir
and W is the work done on the system–the refrigerant

$$\alpha = \frac{Q_2}{Q_1 - Q_2}$$

A refrigerator cannot work without some external work done on the system.i.e.,W cannot be zero or the coefficient of performance cannot be infinite.

Heat Pump

The purpose of a heat pump is to pump heat into a portion of space (the room in a building)when the outside environment is cold.

The Coefficient of Performance

The coefficient of performance (α) of a heat pump is given by

$$\alpha = \frac{Q_1}{W}$$

$$\alpha = \frac{Q_1}{Q_1 - Q_2}$$

Second Law of Thermodynamics

Kelvin-Planck statement

No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.

Clausius statement

No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.
The two statements above are completely equivalent

Reversible and Irreversible Processes

Reversible Processes

A thermodynamic process is reversible if the process can be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe.

Eg: A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.

A process is reversible only if:

1) It is quasi-static i.e., the system in equilibrium with the surroundings at every stage.
2) There are no dissipative factors such as friction, viscosity, etc.

Irreversible Processes

A thermodynamic process is irreversible if the process cannot be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe.

The spontaneous processes of nature are irreversible.

- Eg:The free expansion of a gas
- The combustion reaction of a mixture of petrol and air ignited by a spark .
- Cooking gas leaking from a gas cylinder in the kitchen diffuses to the entire room. The diffusion process will not spontaneously reverse and bring the gas back to the cylinder.

Irreversibility of a process arises due to:

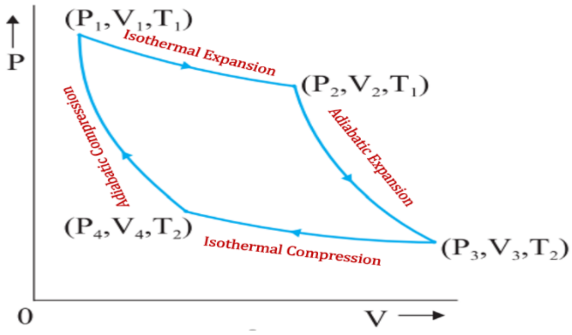
1) Many processes take the system to non-equilibrium states.
2) Most processes involve friction, viscosity and other dissipative effects.

Carnot Engine

Sadi Carnot, a French engineer, developed Carnot engine. Carnot engine is a reversible engine operating between two temperatures T_1 (source) and T_2 (sink).

The working substance of the Carnot engine is an ideal gas.

Carnot cycle



The four processes involved in carnot cycle are

1.Isothermal Expansion
2. Adiabatic Expansion
3. Isothermal Compression
4. Adiabatic Compression

Efficiency of Carnot Engine

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

or

$$\eta = \frac{T_1 - T_2}{T_1}$$

Example

Caculate the efficiency of a heat engine working between steam point and ice point. Can you design an engine of 100% efficiency.

Steam point , $T_1 = 100^0C = 100+273 = 373K$

Ice point , $T_2 = 0^0C = 0+273 = 273K$

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$\eta = \frac{373 - 273}{373}$$

$$\eta = 0.268$$

$$\boldsymbol{\eta = 26.8\%}$$

An ideal engine with $\eta = 1$ or (100%) efficiency is never possible, even if we can eliminate various kinds of losses associated with actual heat engines.

Chapter 13
Kinetic Theory

Ideal gas equation

Gases at low pressures and high temperatures much above that at which they liquefy (or solidify) approximately satisfy a simple relation

$$PV = \mu RT \text{------------(1)}$$

where μ is the number of moles

R is universal gas constant $= 8.314 \text{ J mol}^{-1}\text{K}^{-1}$

T is absolute temperature.

A gas that satisfies this eqn exactly at all pressures and temperatures is defined to be an ideal gas.

The perfect gas equation canalso be written as

$$PV = Nk_BT \text{ -------------(2)}$$

where k_B is Boltzmann constant $=.38 \times 10^{-23}$ J K^{-1}

From above eq , $\frac{PV}{T}$= constant

Real gases approach ideal gas behaviour at low pressures and high temperatures.

At low pressures or high temperatures the molecules are far apart and molecular interactions are negligible. Without interactions the gas behaves like an ideal one.

Boyle's Law

$$PV = \mu RT$$

If we fix μ and T, $\mathbf{PV = Constant}$

$$P \propto \frac{1}{V}$$

i.e., **for a fixed temperature , pressure of a given mass of gas varies inversely with volume. This is the famous Boyle's law.**

Charles' Law

$$PV = \mu RT$$

If we fix P, $\frac{V}{T} = \textbf{constant}$

$$V \propto T$$

i.e., for a fixed pressure, the volume of a gas is proportional to its absolute temperature T (Charles' law).

Dalton's law of partial pressures

Consider a mixture of non-interacting ideal gases μ_1 moles of gas 1, μ_2 moles of gas 2, etc

$$PV = (\mu_1 + \mu_2 + \ldots)\, RT$$

$$P = \mu_1 \frac{RT}{V} + \mu_2 \frac{RT}{V} + \ldots\ldots\ldots\ldots$$

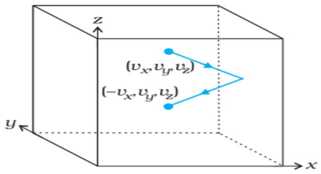
$$P = P_1 + P_2 + \ldots\ldots\ldots\ldots\ldots$$

Thus, the total pressure of a mixture of ideal gases is the sum of partial pressures. This is Dalton's law of partial pressures.

Kinetic Theory of an Ideal Gas

- A given amount of gas is a collection of a large number of molecules that are in random motion.
- At ordinary pressure and temperature, the average distance between molecules is very large compared to the size of a molecule (2 Å).
- The interaction between the molecules is negligible.
- The molecules make elastic collisions with each other and also with the walls of the container .
- As the collisions are elastic , total kinetic energy and total momentum are conserved .

Pressure of an Ideal Gas



The change in momentum of the molecule $= -mv_x - mv_x$
$= -2\,mv_x$

Momentum imparted to wall in the collision $= 2mv_x$

Distanace travelled by the molecule in time $\Delta t = v_x\,\Delta t$
Volume covered by the molecule $= Av_x\,\Delta t$
No of molecules in this volume $= n\,Av_x\,\Delta t$
(n is number density of molecules)
Only half of these molecules move in +x direction
No of molecules $= \frac{1}{2} nA\,v_x\,\Delta t$

The number of molecules with velocity v_x hitting the wall in time Δt

$$= \frac{1}{2} nA\,v_x\,\Delta t$$

The total momentum transferred to the wall

$$Q = (2mv_x)\,(\tfrac{1}{2} nA\,v_x\,\Delta t\,)$$

$$Q = nmAv_x{}^2\,\Delta t$$

The force on the wall, $F = \frac{Q}{\Delta t}$

$$F = nmAv_x{}^2$$

Pressure, $P = \frac{F}{A}$

$$P = nmv_x{}^2$$

All molecules in a gas do not have the same velocity; so average velocity is to be taken

$$P = nm\overline{v_x^2}$$

but $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$

$$\mathbf{P = \frac{1}{3} nm\overline{v^2}}$$

Kinetic Interpretation of Temperature

$$P = \frac{1}{3} nm\overline{v^2}$$

$$PV = \frac{1}{3} nVm\overline{v^2}$$

$n = \frac{N}{V}$, $N = nV$

$$PV = \frac{1}{3} Nm\overline{v^2}$$

where N is the number of molecules in the sample.

$$PV = \frac{2}{3}\ (N\ \frac{1}{2} m\overline{v^2})$$

The quantity in bracket is the average translational kinetic energy of the molecules in the gas.

$$N\ \frac{1}{2} m\overline{v^2}\ = E$$

$$PV = \frac{2}{3}\ E\text{---------------------}(1)$$

Ideal gas equation, $PV = Nk_BT$ -------------------(2)

From eq(1)and (2) $\frac{2}{3}\ E = Nk_BT$

$$E = \frac{3}{2} Nk_BT$$

$$\mathbf{E/N = \frac{3}{2} k_B T}$$

The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas.

Root Mean Square (rms) Speed

$$E/N = \frac{3}{2}k_B T$$
$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$$
$$\overline{v^2} = \frac{3k_B T}{m}$$

The square root of $\overline{v^2}$ is known as root mean square (rms) speed and is denoted by v_{rms}

$$\mathbf{v_{rms} = \sqrt{\frac{3k_B T}{m}}}$$

Degrees f Freedom

The total number of independent ways in which a system can possess energy is called degree of freedom.

A molecule has one degree of freedom for motion in a line.
Two degrees of freedom for motion in a plane.
Three degrees of freedom for motion in space.

Law of Equipartition of Energy

Law of Equipartition of Energy states that, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2}k_B T$

Specific Heat capacities of Monoatomic Gases

The molecule of a monatomic gas has only 3 translational degrees of freedom.

Average energy of a molecule $= 3 \times \frac{1}{2}k_B T = \frac{3}{2}k_B T$

The total internal energy of a mole of such a gas is,

$$U = \frac{3}{2}k_B T \times N_A$$
$$k_B N_A = R$$
$$U = \frac{3}{2}RT$$

Specific heat capacity at constant volume

$$C_V = \frac{dU}{dt}$$
$$= \frac{d}{dT}\left(\frac{3}{2}RT\right)$$
$$\mathbf{C_V = \frac{3}{2}R}$$

For an ideal gas, $C_P - C_V = R$

Specific heat capacity at constant pressure,

$$C_P = C_V + R$$
$$= \frac{3}{2}R + R$$
$$\mathbf{C_P = \frac{5}{2}R}$$

The ratio of specific heats

$$\frac{C_P}{C_V} = \gamma = \frac{\frac{5}{2}R}{\frac{3}{2}R}$$

Adiabatic constant , $\gamma = \frac{5}{3}$

Specific heat capacities of Diatomic Gases

Rigid diatomic molecule

A diatomic rigid rotator has , 3 translational and 2 rotational degrees of freedom.i.e.,

$$\text{Average energy of a molecule} = 5 \ \text{x} \frac{1}{2} k_B T$$

$$= \frac{5}{2} k_B \ T$$

The total internal energy of a mole of such a gas is,

$$U = \frac{5}{2} k_B \ T \ \text{x} \ N_A$$

$$k_B N_A = R$$

$$U = \frac{5}{2} RT$$

Specific heat capacity at constant volume

$$C_V = \frac{dU}{dt}$$

$$= \frac{d}{dT}\left(\frac{5}{2} \ RT\right)$$

$$\mathbf{C_V = \frac{5}{2} R}$$

For an ideal gas, $C_P - C_V = R$ (Mayer's relation)

Specific heat capacity at constant pressure,

$$C_P = C_V + R$$

$$= \frac{5}{2} R + R$$

$$\mathbf{C_P = \frac{7}{2} R}$$

The ratio of specific heats

$$\frac{C_P}{C_V} = \gamma = \frac{\frac{7}{2} R}{\frac{5}{2} R}$$

Adiabatic constant , $\boldsymbol{\gamma = \frac{7}{5}}$

Non Rigid Diatomic Molecule

A non rigid diatomic molecule has , 3 translational , 2 rotational and 1 vibrational degrees of freedom.

(Each vibrational degree of freedom contributes ,2x $\frac{1}{2} k_B T = k_B T$)

$$\text{Average energy of a molecule} = \frac{5}{2} k_B T \ + k_B \ T$$

$$= \frac{7}{2} k_B T$$

The total internal energy of a mole of such a gas is,

$$U = \frac{7}{2} k_B \ T \ \text{x} \ N_A$$

$$k_B N_A = R$$

$$\mathbf{U = \frac{7}{2} RT}$$

Specific heat capacity at constant volume

$$C_V = \frac{dU}{dt}$$

$$= \frac{d}{dT}\left(\frac{7}{2} \ RT\right)$$

$$\mathbf{C_V = \frac{7}{2} R}$$

Specific heat capacity at constant pressure,

$$C_P = C_V + R$$

$$= \frac{7}{2} R + R$$

$$\mathbf{C_P = \frac{9}{2} R}$$

The ratio of specific heats

$$\frac{C_P}{C_V} = \gamma = \frac{\frac{9}{2} R}{\frac{7}{2} R}$$

Adiabatic constant , $\boldsymbol{\gamma = \frac{9}{7}}$

Polyatomic Gases

A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number (f) of vibrational modes.

$$\text{Average energy of a molecule} = \frac{3}{2}k_BT + \frac{3}{2}k_BT + f\,k_BT$$

The total internal energy of a mole of such a gas is,

$$U = \left(\frac{3}{2}k_BT + \frac{3}{2}k_BT + f\,k_BT\right)N_A$$

$$U = (3 + f)k_BTN_A$$

$$k_BN_A = R$$

$$U = (3 + f)RT$$

Specific heat capacity at constant volume

$$C_v = \frac{dU}{dt}$$

$$\mathbf{C_V = (3 + f)R}$$

Specific heat capacity at constant pressure,

$$C_P = C_V + R$$

$$= (3 + f)R + R$$

$$\mathbf{C_P = (4 + f)R}$$

The ratio of specific heats

$$\gamma = \frac{C_P}{C_V} = \frac{(4+f)R}{(3+f)R}$$

Adiabatic constant , $\boldsymbol{\gamma = \frac{(4+f)}{(3+f)}}$

Specific Heat Capacity of Solids

Consider a solid of N atoms, each vibrating about its mean position.

A vibration in one dimension has average energy $= 2x\,\frac{1}{2}k_BT$

$$= k_BT$$

In three dimensions, the average energy $= 3k_BT$

The total internal energy of a mole of solid is,

$$U = 3k_BT \times N_A$$

$$U = 3RT$$

Specific heat capacity $C = \frac{dU}{dt}$

$$= \frac{d}{dT}(3\,RT)$$

$$\mathbf{C = 3R}$$

Specific Heat Capacity of Water

We treat water like a solid.

Average energy of each atom in three dimension$= 3k_BT$

Water molecule has three atoms, two hydrogen and one oxygen.

$$\text{Average energy of Water molecule} = 3 \times 3k_BT$$

$$= 9k_BT$$

The total internal energy of a mole of water is,

$$U = 9k_BT \times N_A$$

$$U = 9RT$$

Specific heat capacity , $C = \frac{dU}{dt}$

$$= \frac{d}{dT}(9\,RT)$$

$$\mathbf{C = 9\,R}$$

Mean Free Path

The mean free path l is the average distance covered by a molecule between two successive collisions.

Chapter 14
Oscillations

Non Periodic Motion
The motion which is non-repetitive .
e.g. rectilinear motion , motion of a projectile.

Periodic Motion
A motion that repeats itself at regular intervals of time is called periodic motion.
e.g. uniform circular motion , orbital motion of planets in the solar system.

Oscillatory Motion
Periodic to and fro motion is called oscillatory motion.
e.g. motion of a cradle , motion of a swing, motion of the pendulum of a wall clock.

Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

Oscillations and Vibration
There is no significant difference between oscillations and vibrations.

- **When the frequency is small, we call it oscillation.**

  e.g.The oscillation of a branch of a tree
- **When the frequency is high, we call it vibration.**

  e.g. The vibration of a string of a musical instrument.

Simple Harmonic Motion
Simple harmonic motion is the simplest form of oscillatory motion.

A particle is said to be in simple harmonic motion ,if the force acting on the particle is proportional to its displacement and is directed towards the mean position.

Mathematical expression for an SHM
If the motion is simple harmonic ,its position can be represented as a function of time.

$$x(t) = A\cos(\omega t + \phi)$$

$$\underset{\substack{\uparrow \\ \text{Displacement}}}{x(t)} = \underset{\substack{\uparrow \\ \text{Amplitude}}}{A} \cos\overbrace{(\underset{\substack{\uparrow \\ \text{Angular frequency}}}{\omega t} + \underset{\substack{\uparrow \\ \text{Phase constant}}}{\phi})}^{\text{Phase}}$$

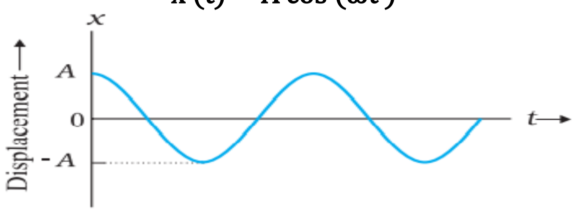
Amplitude
The maximum displacement from the mean poition is called amplitude (A) of oscillation.

Phase
The time varying quantity, **$(\omega t + \phi)$,** is called the phase of the motion.
It describes the state of motion at a given time.

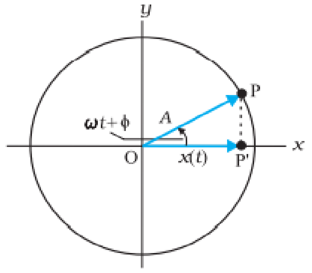
Phase Constant
The constant ϕ is called the phase constant (or phase angle). The value of ϕ depends on the displacement and velocity of the particle at $t = 0$.
The phase constant signifies the initial conditions.

A plot of displacement as a function of time for $\phi = 0$.

$$x(t) = A\cos(\omega t)$$



Simple Harmonic Motion and Uniform Circular Motion



Consider a particle P in uniform circular motion.

The projection of particle along a diameter of the circle is x(t) .

From figure, $\cos(\omega t+\phi)=\frac{x(t)}{A}$

$$x(t) = A\cos(\omega t+\phi) \text{ ----------------(1)}$$

This equation represents a Simple Harmonic Motion.

i. e, the projection of uniform circular motion on a diameter of the circle is in Simple Harmonic Motion.

Velocity in Simple Harmonic Motion

Displacement in SHM is , $x = A\cos(\omega t+\phi)$

Velocity in SHM , $v=\frac{d}{dt}x$

$$v=\frac{d}{dt}[A\cos(\omega t+\phi)]$$

$$v = -\omega A\sin(\omega t+\phi)$$

$$v=-\omega A\sqrt{1-\cos^2(\omega t+\phi)}$$

$$v=-\omega\sqrt{A^2-A^2\cos^2(\omega t+\phi)}$$

$$v=-\omega\sqrt{A^2-x^2} \text{ ------------(2)}$$

Case 1 -At Mean position

$x=0$

$v=\omega\sqrt{A^2-0}$

$$v=\omega A$$

The velocity is maximum At Mean position

Case 2 -At extreme position

$x=A$

$v=\omega\sqrt{A^2-A^2}$

$$v=0.$$

The velocity is minimum at extreme positions.

Acceleration in SHM

Acceleration in SHM , $a=\frac{dv}{dt}$

$$a=\frac{d}{dt}-\omega A\sin(\omega t+\phi)$$

$$a=-\omega A\cos(\omega t+\phi)\ x\ \omega$$

$$a=-\omega^2 A\cos(\omega t+\phi)$$

$$x = A\cos(\omega t+\phi)$$

$$a=-\omega^2 x \text{ -------------(3)}$$

In SHM, the acceleration is proportional to the displacement and is always directed towards the mean position.

Case 1 -At Mean position,

$x=0$

$a=-\omega^2 x$

$$a=0$$

The magnitude of acceleration is minimum at mean position.

Case 2 -At extreme position,

$x=A$

$a=-\omega^2 x$

$$a=-\omega^2 A$$

The magnitude of acceleration is maximum at extreme positions.

Force Law for Simple Harmonic Motion

$$F = ma$$

$$a = -\omega^2 x$$

$$F = -m\omega^2 x$$

$$F = -kx \text{ -------------(4)}$$

where $k = m\omega^2$; $\omega = \sqrt{\frac{k}{m}}$

The force in SHM is proportional to the displacement and its direction is opposite to the direction of displacement. Therefore, it is a restoring force.

Note:

- The centripetal force for uniform circular motion is constant in magnitude, but the restoring force for SHM is time dependent.
- Since the force F is proportional to x such a system is also referred to as a linear harmonic oscillator.

Energy in Simple Harmonic Motion

A particle executing simple harmonic motion has kinetic and potential energies, both varying between the limits, zero and maximum.

Kinetic Energy in Simple Harmonic Motion

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}mv^2$$

$$v = -\omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2(A^2 - x^2)$$

$$K = \frac{1}{2}m\omega^2(A^2 - x^2) \text{ ----------------(5)}$$

Case 1 -At mean position

$$x = 0$$

$$K = \frac{1}{2}m\omega^2(A^2 - 0)$$

$$K = \frac{1}{2}m\omega^2 A^2$$

KE is maximum At Mean position

Case 2 -At extreme position

$$x = A$$

$$K = \frac{1}{2}m\omega^2(A^2 - A^2)$$

$$K = 0.$$

KE is minimum At extreme positions.

Thus the kinetic energy of a particle executing simple harmonic motion is periodic, with period T/2.

Potential Energy in Simple Harmonic Motion

$$U = \frac{1}{2}kx^2$$

$$k = m\omega^2$$

$$U = \frac{1}{2}m\omega^2 x^2 \text{ ----------------(6)}$$

Case 1 -At Mean position

$$x = 0$$

$$U = \frac{1}{2}m\omega^2 x^2$$

$$U = 0$$

PE is minimum At Mean position

Case 2 -At Extreme position

$$x = A$$

$$U = \frac{1}{2}m\omega^2 A^2$$

PE is maximum At extreme positions.

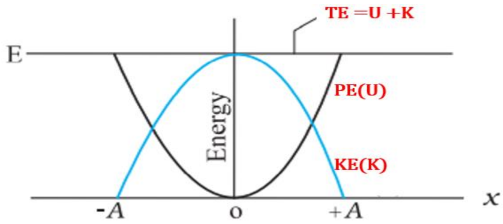
Thus the potential energy of a particle executing simple harmonic motion is also periodic, with period T/2.

The Total Energy in SHM

$$E = U + K$$
$$E = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$E = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$$
$$E = \frac{1}{2}m\omega^2A^2 \text{ ---------------(7)}$$

The total mechanical energy of a harmonic oscillator is a constant or independent of time.

Variation of Potential energy , kinetic energy K and the total energy E with time t for a linear harmonic oscillator



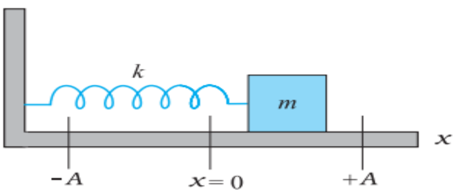
At what position the KE of a simple harmonic oscillator becomes equal to its potential energy?

$$KE = PE$$
$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$$
$$A^2 - x^2 = x^2$$
$$A^2 = 2x^2$$
$$x^2 = \frac{A^2}{2}$$
$$x = \frac{A}{\sqrt{2}}$$

Some Systems Executing Simple Harmonic Motion

There are no physical examples of absolutely pure simple harmonic motion. In practice we come across systems that execute simple harmonic motion approximately under certain conditions.

Oscillations due to a Spring



The small oscillations of a block of mass m fixed to a spring, is fixed to a rigid wall is an example of SHM.
The restoring force F acting on the block is, $F(x) = -k\,x$

k, is called the spring constant.

A stiff spring has large k and a soft spring has small k.

Equation is same as the eqn for force in SHM and therefore the spring executes a simple harmonic motion.

Period of Oscillations of a Spring

Restoring force, $F = -k\,x$

where $k = m\omega^2$

$$\omega^2 = \frac{k}{m}, \quad \omega = \sqrt{\frac{k}{m}}$$

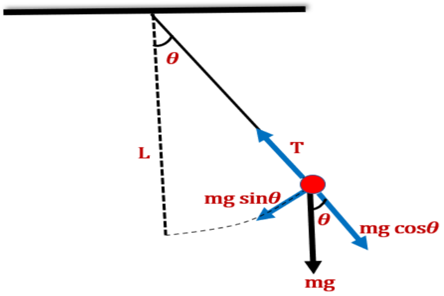
$$\text{Period, } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The Simple Pendulum

A simple pendulum consists of a particle of mass m (bob) suspended from one end of an unstretchable, massless string of length L fixed at the other end.

Period of Oscillations of a Simple Pendulum



$$\tau = -L\,(mg \sin\theta) \text{ --------------(1)}$$

(Where the negative sign indicates that the torque acts to reduce θ.)

For rotational motion we have,

$$\tau = I\,\alpha \text{ -----------------(2)}$$

From eqn (1) and (2)

$$I\,\alpha = -L\, mg \sin\theta$$

$$\alpha = \frac{-mgL}{I}\sin\theta \qquad \text{(since θ is very small, } \sin\theta \approx \theta)$$

$$\alpha = \frac{-mgL}{I}\theta \text{ ---------------(3)}$$

Acceleration of SHM , $a = -\omega^2 x$ -------------------(4)

Comparing eqns (3) and (4)

$$\omega^2 = \frac{mgL}{I}$$

$$I = mL^2$$

$$\omega^2 = \frac{mgL}{mL^2}$$

$$\omega^2 = \frac{g}{L}$$

$$\omega = \sqrt{\frac{g}{L}}$$

Period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}}$

$$\mathbf{T = 2\pi\sqrt{\frac{L}{g}}}$$

Example

What is the length of a simple pendulum, which ticks seconds (seconds pendulum) ?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{l}{g}$$

$$L = \frac{T^2 g}{4\pi^2}$$

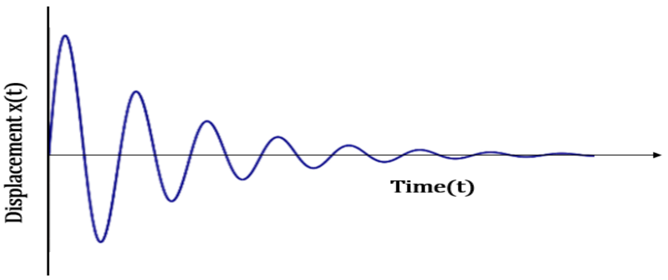
For seconds pendulum ,T = 2s

$$L = \frac{2^2 \times 9.8}{4 \times 3.14^2} = 0.994 \approx 1\text{m}$$

Damped Simple Harmonic Motion

Periodic oscillations of gradually decreasing amplitude due to external forces, are called dampes simple harmonic motion

Displacement as a function of time in damped harmonic oscillations



Free Oscillations

If an oscillator is displaced and released, it begins to oscillate with its natural frequency (ω). Such oscillations in the absence of an external force are called free oscillations. Due to dissipative forces, the free oscillations cannot be sustained.

E.g, A person swinging in a swing without anyone pushing it .
A simple pendulum, displaced and released.

Forced Oscillations

If a periodic force is applied to a free oscillator to sustain its oscillations, it will oscillate with the frequency of driving force (ω_d). Such oscillations are called forced or driven oscillations.

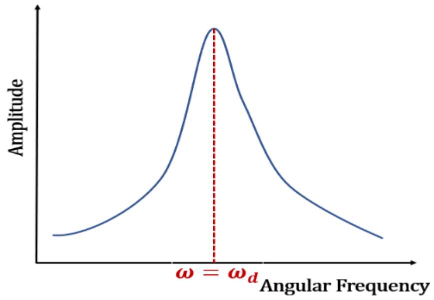
E.g.,while swinging in a swing if you apply a push periodically by pressing your feet against the ground, you can maintain oscillations and can increase amplitude.

Resonance

The phenomenon of increase in amplitude when the driving frequency(ω_d) is equal to the natural frequency (ω) of the oscillator is called resonance.

At resonance, $\omega = \omega_d$

The variation of amplitude of a forced oscillator with angular frequency at resonance



Some daily life phenomena which involve resonance:

- You can swing to greater heights ,if the rhythm of pushing against the ground is equal to the natural frequency of the swing.
- At Tacoma Narrows Bridge at , Washington, USA winds produced a pulsating resultant force in resonance with the natural frequency of the structure. This caused a steady increase in the amplitude of oscillations until the bridge collapsed.
- The marching soldiers break steps while crossing a bridge to avoid resonance condition.
- Aircraft designers make sure that none of the natural frequencies at which a wing can oscillate match the frequency of the engines in flight.
- In an earthquake, short and tall structures remain unaffected while the medium height structures fall down. This happens because the natural frequencies of the short structures happen to be higher and those of taller structures lower than the frequency of the seismic waves.

Chapter 15
Waves

Waves
The patterns, which move without the actual physical transfer or flow of matter as a whole, are called waves. The waves we come across are mainly of three types:

(a) Mechanical waves,
(b) Electromagnetic waves and
(c) Matter waves.

Transverse and Longitudinal Waves
Mechanical waves can be transverse or longitudinal depending on the relationship between the directions of vibrations of particles in the medium and that of the propagation of wave.

Transverse waves
In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation.

- They travel in the form of crests and troughs.
- Transverse waves can be propagated only in solids and strings, and not in fluids.
- E.g, Waves on a stretched string,

Longitudinal waves
In longitudinal waves the constituents of the medium oscillate along the direction of wave propagation.

- They travel in the form of compressions and rarefactions.
- Longitudinal waves can propagate in all elastic media,i.e,solids,liquids nd gases.
- E.g, sound waves, vibrations in a spring.

Travelling or Progressive Wave
A wave, transverse or longitudinal, is said to be travelling or progressive if it travels from one point of the medium to another.

Displacement Relation in a Progressive Wave along a String (transverse wave)
A progressive wave travelling along the positive direction of the x-axis can be represented as

$$y(x,t) = a\sin(kx - \omega t + \phi)$$

A progressive wave travelling along the negative direction of the x-axis can be represented as

$$y(x,t) = a\sin(kx + \omega t + \phi)$$

$$\overbrace{y(x,t)}^{\text{Displacement}} = \overbrace{a}^{\text{Amplitude}} \sin(\overbrace{\underbrace{kx}_{\substack{\text{Angular}\\ \text{Wave}\\ \text{Number}}} - \underbrace{\omega t}_{\substack{\text{Angular}\\ \text{Frequency}}} + \underbrace{\phi}_{\substack{\text{Initial}\\ \text{Phase}\\ \text{Angle}}}}^{\text{Phase}})$$

Propagation Constant or Angular Wave Number
Propagation constant or Angular Wave Number is defined as

$$k = \frac{2\pi}{\lambda}$$

Its SI unit is radian per metre or rad m^{-1}

Angular Frequency
Angular Frequency of a wave is given by

$$\omega = \frac{2\pi}{T}$$

Its SI unit is rad s^{-1}

From this equation, $\mathbf{T} = \frac{2\pi}{\omega}$

Example

A wave travelling along a string is described by,
y(x, t) = 0.005 sin (80.0 x – 3.0 t), in which the numerical constants are in

SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate

(a) the amplitude,
(b) the wavelength,
(c) the period and frequency of the wave.
(d) Calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s ?

Answer

y(x, t) = 0.005 sin (80.0 x – 3.0 t)

The general expression for a travelling wave is

y (x,t) = a sin (kx – ωt + ϕ)

Comparing these equations

(a) Amplitude , **a=0.005m**

(b) k=80 rad m^{-1}

but , k= $\frac{2\pi}{\lambda}$

$\frac{2\pi}{\lambda} = 80$

$\lambda = \frac{2\pi}{80}$ =0.0785 m

$\boldsymbol{\lambda = 7.85\ cm}$

(c) ω=3

but, $\omega = \frac{2\pi}{T}$

$\frac{2\pi}{T} = 3$

$T = \frac{2\pi}{3} = \mathbf{2.09\ s}$

Frequency, ν = 1/T
=1/2.09 **= 0.48 Hz**

(d) y(x, t) = 0.005 sin (80.0 x – 3.0 t)
x = 30.0 cm =0.3m
t = 20 s
y(x, t) = 0.005 sin (80.0 x 0.3 – 3.0 x20)
= (0.005 m) sin (–36)
= 5 mm

The Speed of a Travelling Wave

Consider a wave propagating in positive x direction with initial phase ϕ =0

y (x, t) = a sin (kx – ωt)

(kx – ωt) = constant

$\frac{d}{dt}$(kx – ωt) = 0

$k\frac{dx}{dt} - \omega\frac{dt}{dt} = 0$

$\frac{dx}{dt} = \frac{\omega}{k}$

$\mathbf{v = \frac{\omega}{k}}$

$\omega = 2\pi\nu$, k=$\frac{2\pi}{\lambda}$

$v = \frac{2\pi\nu}{\frac{2\pi}{\lambda}}$

$\mathbf{v = \nu\lambda}$

This is a general relation valid for all progressive waves.

Speed of a Transverse Wave on Stretched String

The speed of transverse waves on a string is determined by two factors,

(i) the linear mass density or mass per unit length, μ, and

(ii) the tension T

$$v=\sqrt{\frac{T}{\mu}}$$

The speed of a wave along a stretched ideal string does not depend on the frequency of the wave.

Speed of a Longitudinal Wave(Speed of Sound)

The longitudinal waves in a medium travel in the form of compressions and rarefactions or changes in density, ρ.

- **The speed of propagation of a longitudinal wave in a fluid**

$$v=\sqrt{\frac{B}{\rho}}$$

B= the bulk modulus of medium, ρ = the density of the medium

- **The speed of a longitudinal wave in a solid bar**

$$v=\sqrt{\frac{Y}{\rho}}$$

Y =Young's modulus , ρ=density of the medium,

- **The speed of a longitudinal wave in an ideal gas**

Case1 -Newtons Formula

Newton assumed that, the pressure variations in a medium during propagation of sound are isothermal.

$$v=\sqrt{\frac{P}{\rho}}$$

This relation was first given by Newton and is known as **Newton's formula.**

Case 2- Laplace correction to Newton's formula.

Laplace that the pressure variations in the propagation of sound waves are so fast that there is little time for the heat flow to maintain constant temperature. These variations, therefore, are adiabatic and not isothermal.

$$v=\sqrt{\frac{\gamma P}{\rho}}$$

This modification of Newton's formula is referred to as the **Laplace correction.**

$\gamma = \frac{C_P}{C_V}$, For air $\gamma = \frac{7}{5}$.

The speed of sound in air at STP = 331.3 m s^{-1}

The Principle of Superposition of Waves

When two or more waves overlap, the resultant displacement is the algebraic sum of the displacements due to each wave.

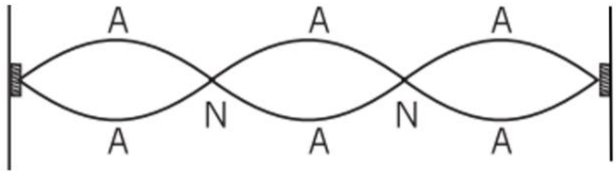
let y_1(x, t) and y_2 (x, t) be the displacements individual waves, then resultant displacement y (x, t) is,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

Standing Waves and Normal Modes

Standing Waves

The interference of two identical waves moving in opposite directions produces standing waves.



Wave travelling in the positive direction of x-axis

$$y_1(x, t) = a \sin(kx - \omega t)$$

Wave travelling in the negative direction of x-axis

$$y_2(x, t) = a \sin(kx + \omega t)$$

By the principle of superposition

$$y(x, t) = y_1(x, t) + y_2(x, t) = a \sin(kx - \omega t) + a \sin(kx + \omega t)$$

$$y(x, t) = (2a \sin kx) \cos \omega t$$

This equation represents a standing wave, a wave in which the waveform does not move.

Amplitude of wave , $A = 2a \sin kx$.

Nodes and Antinodes

The positions of zero amplitude in a staning wave are called nodes and the positions of maximum amplitude are called antinodes.

Condition for Nodes

At nodes,the amplitude of standing wave is zero

$$2a \sin kx = 0$$

$$\sin kx = 0$$

$$kx = n\pi, \text{ for } n = 0, 1, 2, 3, ..$$

$$\text{But } k = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda} x = n\pi$$

$$x = n\frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, 3, ..$$

i.e., nodes are formed at locations x=0, $\frac{1\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}$ **,... ......**

Condition for Antinodes

At antinodes, the amplitude of standing wave is maximum.

$$2a \sin kx = \text{maximum}$$

$$\sin kx = \pm 1$$

$$kx = (n + \tfrac{1}{2})\pi, \text{ for } n = 0, 1, 2, 3, ..$$

$$\text{but }, k = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda} x = (n + \tfrac{1}{2})\pi$$

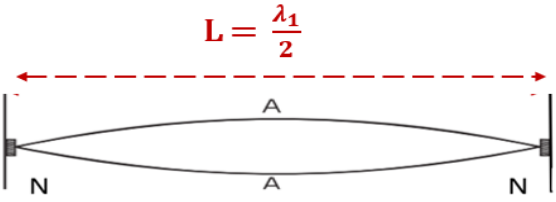
$$x = (n + \tfrac{1}{2})\frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, 3, ..$$

i.e., antinodes are formed at locations x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ **,.........**

(1)Standing waves in a Stretched String fixed at both the ends

Fundamental mode or the first harmonic

The oscillation mode with n=1 , the lowest frequency is called the fundamental mode or the first harmonic.



$L = \frac{\lambda_1}{2}$

$\lambda_1 = 2L$

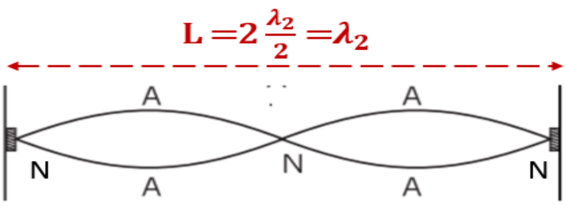
But $v = \nu\lambda$, $\nu = \frac{v}{\lambda}$

Frequency, $\nu_1 = \frac{v}{\lambda_1}$

$\nu_1 = \frac{v}{2L}$ ----------(1)

The second harmonic

The second harmonic is the oscillation mode with n = 2.



$L = 2\frac{\lambda_2}{2} = \lambda_2$

$\lambda_2 = L$

Frequency, $\nu_2 = \frac{v}{\lambda_2}$

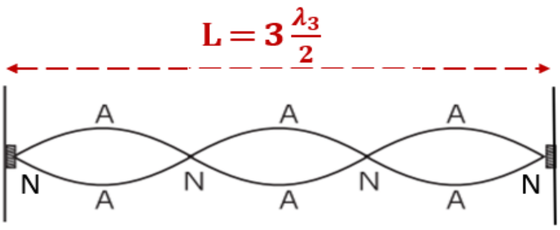
$\nu_2 = \frac{v}{L}$

$\nu_2 = 2\frac{v}{2L}$ -------------(2)

$\nu_2 = 2\nu_1$

The Third Harmonic

The third harmonic is the oscillation mode with n = 3.



$L = 3\frac{\lambda_3}{2}$

$\lambda_3 = \frac{2L}{3}$

Frequency, $\nu_3 = \frac{v}{\lambda_3}$

$\nu_3 = \frac{v}{\frac{2L}{3}}$

$\nu_3 = 3\frac{v}{2L}$ --------------(3)

$\nu_3 = 3\nu_1$

and so on.

$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$

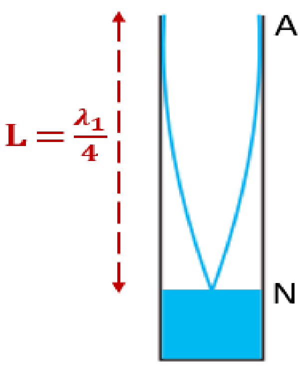
Thus all harmonics are possible in a stretched string fixed at both the ends.

(2) The modes of vibration in a closed pipe (system closed at one end and the other end open).

Eg: Resonance Column(Air columns such as glass tubes partially filled with water) .

Fundamental mode or the first harmonic

The oscillation mode with n=0 , fundamental mode or the first harmonic.



$L = \frac{\lambda_1}{4}$

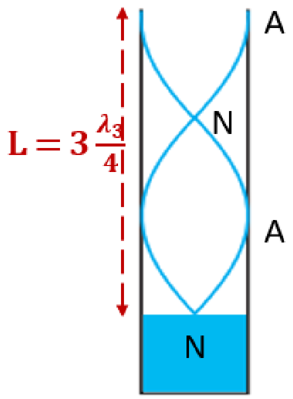
$\lambda_1 = 4L$

Frequency, $\nu_1 = \frac{v}{\lambda_1}$

$\nu_1 = \frac{v}{4L}$ ------------(1)

The Third Harmonic

The Third harmonic is the oscillation mode with n = 1.



$L = 3\frac{\lambda_3}{4}$

$\lambda_3 = \frac{4L}{3}$

Frequency, $\nu_3 = \frac{v}{\lambda_3}$

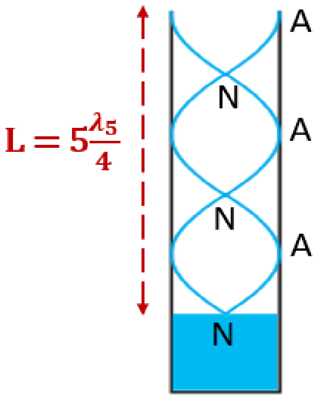
$\nu_3 = \frac{v}{\frac{4L}{3}}$

$\nu_3 = 3\frac{v}{4L}$ --------------(2)

$\nu_3 = 3\nu_1$

The Fifth Harmonic

The Fifth harmonic is the oscillation mode with n = 2.



$L = 5\frac{\lambda_4}{4}$

$\lambda_4 = \frac{4L}{5}$

Frequency, $\nu_5 = \frac{v}{\lambda_5}$

$\nu_5 = \frac{v}{\frac{4L}{5}}$

$\nu_5 = 5\frac{v}{4L}$ -------------(3)

$\nu_5 = 5\nu_1$

And so on

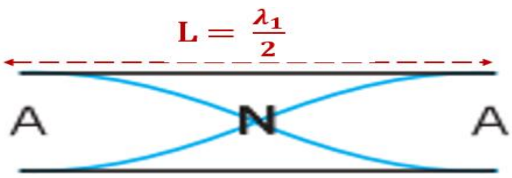
$\nu_1 : \nu_3 : \nu_5 = 1 : 3 : 5$

Thus only odd harmonics are possible in a closed pipe.

(3)The modes of vibration in a an open pipe (system open at both ends). Eg: Flute

Fundamental Mode or The First Harmonic

The oscillation mode with n=1 , the lowest frequency is called the fundamental mode or the first harmonic.



$L = \frac{\lambda_1}{2}$

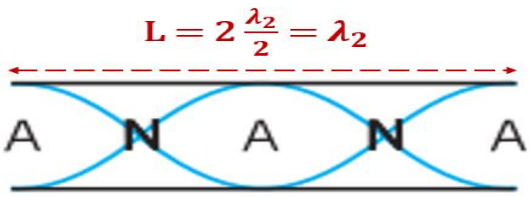
$\lambda_1 = 2L$

Frequency, $\nu_1 = \frac{v}{\lambda_1}$

$\nu_1 = \frac{v}{2L}$ ------------(1)

The Second Harmonic

The second harmonic is the oscillation mode with n = 2.



$L = 2\frac{\lambda_2}{2}$

$\lambda_2 = L$

Frequency, $\nu_2 = \frac{v}{\lambda_2}$

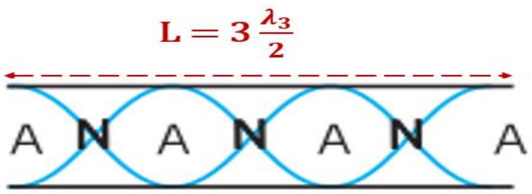
$\nu_2 = \frac{v}{L}$

$\nu_2 = 2\frac{v}{2L}$ ------------(2)

$\nu_2 = 2\nu_1$

The Third Harmonic

The third harmonic is the oscillation mode with n = 3.



$$L = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

Frequency, $\nu_3 = \frac{v}{\lambda_3}$

$$\nu_3 = \frac{v}{\frac{2L}{3}}$$

$$\nu_3 = 3\frac{v}{2L} \text{------------(3)}$$

$$\nu_3 = 3\nu_1$$

and so on.

$$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$$

Thus all harmonics are possible in an open pipe.

So open pipes are preferred over closed pipes in musical instruments.

Beats

The periodic variations(wavering) of sound intensity when two waves of nearly same frequencies and amplitudes travelling in the same direction, are superimposed on each other is called beats.

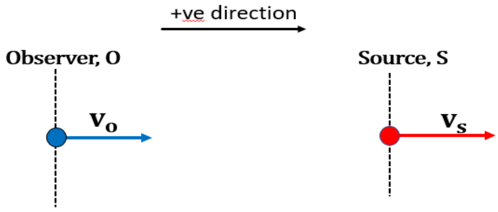
These wavering of sound is also called waxing and waning.

If ν_1 *and* ν_2 are the frequencies of superposing waves, the beat frequency

$$\nu_{beat} = \nu_1 - \nu_2$$

Doppler Effect

The apparent change in the observed frequency of a wave when the source and the observer moves relative to the medium is called Doppler Effect.



Consider the source and observer are moving with velocity v_s and v_o in the same direction velocity of sound, v_s .

Let ν_s be the frequency of sound.

The apparent frequency of sound heard by the observer is,

$$\nu_o = \nu_s \left(\frac{v + v_o}{v + v_s}\right)$$